

# THE MONIST

## HUXLEY IN HIS EPOCH<sup>1</sup>

TO forestall misinterpretation of aught I may say later, let me confess at the outset that Huxley is a man after my own heart. Constructive and thoroughly competent in his specialty; an eager, intrepid spirit to whom nothing human was alien; rigid when questions of principle arose, keenly conscious that here *suaviter in modo* must always play second fiddle to *fortiter in re*; absolutely above board, evading none of the disagreeables which, thanks to flaccid convention, camp on the trail of outspoken honesty; an incarnation of kindness to those in real need of help; a scourge of pretention, especially of ignorant pretention; a crusader for practicable reform, well aware, however, that reform cannot prosper unless practicable tradition be retained; of extraordinary lucidity as teacher, writer, and speaker; in a word, the greatest all-round human being in that remarkable group who, beginning life with *Sartor Resartus*, brought up at *Dover Beach*, took stock of resources in *Fifine at the Fair*, and passed out, still sure of the rightness of the world, *Crossing the Bar* their sufficient confession.<sup>2</sup> In sum, then, a man representative of an epoch; not an "eccentric" or "original," no head-strong sectary, no lonely soul prinking in "freedom" from contemporary "limitations"; on the contrary, a thoroughly typical person, never more than in his allegiance to the half-truths he had from Carlyle, from a straitened inter-

<sup>1</sup> A Centennial Address, delivered before the Research Club of the University of Michigan, April, 1925.

<sup>2</sup> Huxley's debt to Carlyle, his affinities with Matthew Arnold, his admiration for Browning and Tennyson are matters of record.

pretation of Hume, and from prevalent Utilitarianism,—a symptomatic, if curious, medley. Thus, to appreciate him, we must hark back to a period so different from our own as to have become almost a tale that is told.

## I

Huxley died just a generation ago (1895), in the same year as Cayley, Ludwig, and Pasteur; Helmholtz and Hertz, Oliver Wendell Holmes, Robert Louis Stevenson, and Walter Pater had passed away the year before. When we consider what these names represent, we feel as if an aeon had elapsed since. For this was the time of the Battle of the Malakand Pass, of Stambuloff's murder, of the retirement of Tricoupis from public life. We may be pardoned if we pull a grimace, asking hazily, What was the Battle of the Malakand Pass? Who were Stambuloff and Tricoupis? Only when we recall Cleveland's intervention over the boundary dispute between Venezuela and Britain does a nod of recognition restore our comfort; and, even here, recognition may revert to puzzlement if it be suggested that Cleveland was "used" by a somewhat sardonic blackguard, one Castro! Hence, the delicacy, not to say difficulty, of my task this evening. I must try "to unravel the precious web of time without tangling it"; in other words, I must launch into the perspective of ideas, dodging unprolific "facts" and, greatly daring, make play with Tyndall's favorite instrument, the "scientific use of imagination"—a rare tool for spirits like Huxley, apt to be pernicious in the hands of common folk. For, the displacements of the past century have been so unique that a vigorous effort of "imagination" is necessary to recapture the outlook which, taken as matter of course by our aged

grandfathers, had become the chief subject of debate in the strenuous maturity of our fathers. Moreover, England, and Huxley with England, stepped into them, as it were, from a peculiar background determined by an intellectual and social tradition quite different from that of other European countries, France and Germany conspicuously. If we miss this we miss everything. Accordingly, it may serve the present purpose best to set out from this background, and with special reference to the position of the natural sciences. It were superfluous to recite the details assembled so admirably in the *Life and Letters*. Let it suffice for the moment that we attempt to experience the significant impact of a story high incredible in these days of "the organization of science."

Born in a London suburb, son of an assistant master in what the English call an "adventure" school,—that is, an "establishment" dependent upon the oversight of the head and owner—circumstances so conspired that Huxley never had any formal education worth mention. Left practically to himself during early adolescence, he read omnivorously, being, as he says, "a dreamer and speculator of the first water, well endowed with that splendid courage in attacking any and every subject which is the blessed compensation of youth and inexperience." He "devoured with avidity" Hamilton, *On the Philosophy of the Unconditioned* (1829), finding this *Edinburgh Review* paper by mere accident, much in the same way as Nietzsche lit upon Schopenhauer at a bookstall and, curiously enough, to similar effect. He never outgrew his reaction to Hamiltonianism and, I suspect, died unaware that "Hamilton's reputation has not withstood the test of time."<sup>3</sup> Guizot's *History of Civilisation* attracted; but Carlyle, who sent Huxley to German, left the profoundest impress. These fine, free divagations gave place to more systematic ways in

<sup>3</sup> Cf. W. R. Sorley, *A History of English Philosophy*, p. 234 (1921).

1842 when, after drab experience as an "apothecary's assistant" in the warrens of East London, and some pre-medical training at Sydenham College, he joined Charing Cross Hospital, to be stimulated by the instruction of Wharton Jones. Three years later,—having discovered "Huxley's sheathe" meantime,—he took high honors in anatomy and physiology at the London M. B. examination. The crisis, destined to transform him into an investigator, arose soon. Following the example of Darwin (1831), J. D. Hooker (1839), and Edward Forbes (1841)—all medical students, be it observed—he accepted an appointment as naval "assistant surgeon," and sailed aboard H. M. S. *Rattlesnake* (Dec. 3, 1846), bound for a survey of the Australian Barrier Reef and New Guinea waters, to be gone almost four years. I may not poach upon the preserves of my colleague here,<sup>4</sup> save to remark that the paper on the *Medusae*, transmitted from the Antipodes, brought Huxley election to the Royal Society when twenty-six and, a year later, award of the Royal Medal. These recognitions set him finally on the path of research. Our good neighbor, Toronto, was blind enough to pass him over when candidate for her chair of "Natural History"; he was rebuffed similarly by Aberdeen, Queen's College, Cork, and King's College, London. For the moment, all roads even to bare livelihood seemed shut. Eventually, in 1854, he obtained a Professorship at the Museum of Practical Geology, London, in succession to Edward Forbes, at a salary of \$1,000 a year; higher academic preferment, of the kind consecrated by English tradition, was proffered too late to be acceptable.

What an extraordinary state of affairs, when judged from our contemporary standpoint! Here is a youth who had neither art nor part in the national education of his native land—an "outsider" as concerns the great schools

<sup>4</sup> The companion Address, on Huxley's scientific achievement, was given by Dr. E. C. Case, Professor of Palaeontology.



and universities—who labored under tremendous, almost inconceivable handicap, as we should now think, because he never sat at the feet of any scientific leader; who became, as the somewhat contemptuous phrases were, an “apothecary’s assistant,” and “hospital-walker”; who, *faute de mieux*, betook himself to serve as a “donkey-frigate” officer, when, according to modern conventions, he ought to have been “doing graduate work”; who became, nevertheless, almost at a single leap, of the elect of English science; yet who remained throughout life apart from the universities of the Empire, not without early appeal on his part for even modest recognition! This is the “impact” I wish you to get and, in seeking some explanation, to sense the peculiar circumstances which account for so much, not in Huxley alone, but in many others. His own comment, as we shall see in the sequel, is full of relevance. “England can show now, as she has been able to show in every generation since civilization spread over the West, individual men who hold their own against the world, and keep alive the old tradition of her intellectual eminence. But in the majority of cases, these men are what they are in virtue of their native intellectual force, and of a strength of character which will not recognize impediments. They are not trained in the courts of the Temple of Science, but storm the walls of that edifice in all sorts of irregular ways, and with much loss of time and power, in order to obtain their legitimate positions.” In his Memorial Lecture (Oct., 1898), Virchow noted this freedom “from the formalism of the schools” as among Huxley’s manifest advantages. Both give a cue to the peculiar conditions of science in England, which we may well follow forthwith.

So early as 1808 John Playfair had said, such was the backwardness of English mathematics, that “we shall hardly reckon a dozen . . . who can read the “*Mécanique*

céleste" with any tolerable facility."<sup>5</sup> Thirteen years later Cuvier took a different view.<sup>6</sup> Nevertheless, Brewster's review (1830), which blazed the trail for the British Association, asserts "that within the last fifteen years not a single discovery or invention of prominent interest has been made in our colleges, and that there is not one man in all the eight universities of Great Britain who is at present known to be engaged in any train of original research."<sup>7</sup> Native detractors and foreign apologists were both right. A glance at the facts may suffice to make this plain. Time fails me to condescend upon details; suffice it to say that, between 1744 and 1843, we find the following, among other, important names: Landen, Canton, Cavendish, Maskeleyne, Priestley, John Dawson, Benjamin Thompson, F. W. Herschel, Nicholson, Wells, Ivory, Dalton, Wollaston, William Smith, Young, Robert Brown, Groombridge, Prout, Bewick, Malthus, Kater, Davy, Sabine, Faraday, George Green, Hugh Miller, John Lindley, De Morgan, Boole, Joule; thirty in all. Now, note that eighteen (sixty per cent) never matriculated at any university. The other twelve had some connection with Cambridge, St. Andrews, Aberdeen, Edinburgh, or London (where there was no university!). At this, however, one-half were medical students and, in the state of medical education then, could not be termed university men—(another twenty per cent, making eighty per cent in all!). Nay, even the "university men" derived little from their formal studies. For example, Maskeleyne pursued "divinity" at Cambridge; George Green entered the same university at forty, his reputation already won, and was chosen a Fellow at forty-seven; Ivory left St. Andrews to become a flax-spinner; while Malthus derived impulse, not from college work, but from his father's friend, Rousseau. Turning to the others;

<sup>5</sup> *Edinburgh Review*, ii, 279f.

<sup>6</sup> *Eloges historiques*, iii, 79 (on Sir Joseph Banks).

<sup>7</sup> *Quarterly Review*, xliii, 327.

what a queer job lot they seem, out of every vocation under heaven, science excepted! A land agent or factor, a dissenting preacher, an immigrant Yankee quasi-official, a literary hack, an immigrant Hanoverian organist, an officer of a provincial "literary and philosophical" society, a surveyor (civil engineer), a wood engraver, a couple of soldiers, a surgeon's apprentice, an apothecary's assistant (like Huxley), a journeyman bookbinder, a stone mason, a market gardener, a teacher in a "venture" school (like Huxley's father). Notice, too, how great ability had to be content with small reward. At forty Malthus got so far as to become a "professor" in the Honorable East India Company's school at Haileybury; De Morgan, "a career at his own university being closed against him," enters Lincoln's Inn, and never climbs higher than a chair at a new and struggling London college where John Lindley was his colleague; in the same way, Boole is thirty-four ere he receives a professorship at the remote Irish college where he was destined to die; and, going farther back, Dalton is sixty-six before Oxford discovers him! The inference is overwhelming. Cuvier was right when he referred to the scientific distinction of England, Brewster was right when he drew attention to the utter lack of organization, and to the aloofness of the ancient universities. This, then, reveals the peculiar background of English science out of and into which Huxley stepped. It had everything to do especially with the *temper* of his life-work. For, consider the contrast by comparison with France and Germany.

Existing for years as an informal, private association, decorated by such names as Descartes, Gassendi, and Pascal, the Academy of Sciences received official *status* in 1666, thanks to the foresight of Colbert. Despite vicissitudes, particularly under the Revolution, it maintained itself, due to Carnot's organization of the *Institut National* (1795), never with a more brilliant galaxy than on its

reconstitution in 1816. Naturally, too, the example of Paris spread to other centers. Now, whatever the defects of Academies (fairly plain), conspicuous merits abound, chiefly opportunity for interchange of opinion and for co-operation. There can be little doubt that French scientific leadership, supreme in the eighteenth century, is traceable to the academicians; Newton remained a solitary "peak in Darien," *Newtonianisme* was the product of Gallic socialization, given popular vogue by Voltaire. And one may as well remark in passing that precisely the same happened to be true of Locke and *Sensualisme*. The English thinker wrought by himself, the system elaborated from his thought is French.

"It would be superfluous to dwell on the value for other countries of the example and experience of Germany. There is no people which has given so much thought and pains to the development of its university system as the Germans"—so wrote that "good European," James Bryce, in 1885.\* From Prague (1348) in the beginning, through Freiburg (1457) and Tübingen (1477) in the era of Humanism, and Göttingen (1734) which always stood for *Lehr und Lehrfreiheit*, down to Berlin (1810) most influential of all, the German universities have initiated and brought to completion greater works dependent upon scientific co-operation than those of any country. Here, again, as in France, there were manifest abuses, with one overwhelming compensation—complete organization of *Wissenschaft* all round. Hence, as Fichte said, "in the academic teacher *Wissenschaft* is to speak, not the teacher himself." English individualism and idiosyncrasy lay outside the bounds of practical politics to all intents and purposes. Darwin worked at Down, almost a recluse, *Darwinismus* was "made in Germany." In effect, then, such

\* Cf. his Preface to the translation, by my old teacher, John Hutchison, of J. Conrad's *The German Universities for the last Fifty Years* (1885). This book describes a situation which some of our younger colleagues may have forgotten.

a career as Huxley's was improbable in France, almost inconceivable in Germany. Not without evident consequences, one of which was Huxley's activity as a "public character" after 1870. In a word, certain most definite circumstances lay behind him, rendering him a veritable phenomenon; this situation it is that justifies my title—"Huxley in His Epoch."

## II

As usual, these circumstances were of three types; one cannot disentangle them, because each reacts upon the others. Definite social, intellectual, and institutional folkways had been well worn. Socially, England developed and kept differences of class; general esteem set the landed gentry on a pedestal, to be like unto them was the desirable end of life. Accordingly, practical affairs loomed largest, and took precedence over even the intellectual outlook or interest deemed "proper" by the directing group. So much so that, as late as 1868, an eminent German immigrant could assert: "Nowhere has the tradition of classical learning been handed down more faithfully from one generation to another than in England; nowhere has its generous spirit more thoroughly pervaded the minds of statesmen, poets, artists, and moulded the character of that large and important class of independent and cultivated men, without which this country would cease to be what it has been for the last two centuries, a *res publica*, a commonwealth, in the best sense of the word." Small wonder, then, it did not enter into the calculations of governing folk to produce scholars and investigators. It was desirable, eminently desirable, to produce "educated men, furnished with so much of liberal culture as would enable

them to maintain their position in life and society, or to succeed better in any practical pursuit in which they might engage,"\* particularly service of the State. Hence, representative institutions bent to these ideals insensibly and inevitably. Moreover, the universities had become subjects of a negative *and* a positive condition by reason of their close alliance with the Church of England. Non-conformist ability found itself out in the cold, while the better academic minds were enlisted by the emoluments and social consideration attendant upon ecclesiastical preferment. Thus, exclusion and inducement led to unconscious selection and stress.<sup>10</sup> Notwithstanding, it would be a mistake to allege, as some have done (Huxley not excepted), that natural science received no attention whatsoever. Take even Oxford, the less favorable example. As early as 1400, the Statutes of New College provide for study of astronomy and medicine, those of Magdalen (1479) for instruction in "natural philosophy." Nor may we forget that Linacre, of All Souls, was instrumental in founding the Royal College of Physicians; that Sydenham, of Christ Church, friend of the Oxford physician-philosopher Locke, quite deserved his *soubriquet*, "the English Hippocrates"; or that the Royal Society, Boyle's "Invisible College" (1646), owed its existence to *alumni* of the ancient universities. Still, it remained true till the latter half of the nineteenth century that "original research in England is either the privilege of persons of fortune, or it is performed in the intervals of business or professional labor other than education."<sup>11</sup> The thirty names cited above furnish abundant proof. To render the matter more realistic, bringing it down to Huxley's maturity, let me add two others from boyhood acquaintance: for the "per-

\* Max Müller's Inaugural Address at Oxford; cf. *Chips from a German Workshop*, vol. iv, pp. 1ff. The italics are mine.

<sup>10</sup> In 1861 but 27 per cent of the M. A.'s were laymen; cf. J. E. T. Rogers, *Education in Oxford*, p. 6 (London, 1861).

<sup>11</sup> Cf. C. E. Appleton in the *Fortnightly Review*, Oct., 1876.

sons of fortune," my kinsman George J. Romanes; for those who "performed in the intervals of business," my father's colleague in the Bank of Scotland, Robert Gray, an authority on ornithology.<sup>12</sup> As Max Müller has it once more: "There is no career in England at the present moment for scholars and students. No father could honestly advise his son, whatever talent he might display, to devote himself exclusively to classical, historical, or physical studies. The few men who still keep up the fair name of England by independent research and new discoveries in the fields of political and natural history, do not always come from our universities; and unless they possess independent means, they cannot devote more than the leisure hours, left by their official duties in Church or State, to the prosecution of their favorite studies."<sup>13</sup>

To make a long story short, in Huxley's day few, if any, had come to see that scholarship, much less science, possess worth quite without reference to "educational" or other institutions. Even in the universities a very small minority realized that these societies are under bonds to increase as well as to diffuse knowledge. Theology and its supposititious appanages enjoyed a preference over equally important pursuits because, thanks to the "lie of the land," they were avenues to livelihood and esteem. There can be no doubt that these things must be held accountable for much of the *temper* in which Huxley conducted his life-work—for unfortunate no less than fortunate traits and consequences. His engrained suspicion of the ancient universities till the bitter end, long after they had held out the olive branch again and again, intimates a great deal. Three years before his death, he banters his daughter about the education of her son thus: "Oxford always represents English opinion in all its extremes better than Cambridge. Cambridge better for doc-

<sup>12</sup> Cf. *The Birds of the West of Scotland* (1871).

<sup>13</sup> *L. c.*, p. 71.



tors (i. e., physicians). Oxford for architects, poets, painters, and all that sort of cattle." <sup>14</sup> Scarcely a genial way to put it! On the contrary, here is the more fortunate result. "'Authorities,' 'disciples,' and 'schools' are the curse of science, and do more to interfere with the work of the scientific spirit than all its enemies." <sup>15</sup> The zeal of the house of science had eaten him up, the more that the iron had entered into his soul, thanks to the peculiar disposition of affairs in his native land. We have just heard what he said in old age; listen to him as he reviews his youth. "When you and I were youngsters," he writes to Hooker, "we thought it the great thing to exorcise the aristocratic flunkeyism which reigned in the Royal Society." Ever alert, he now detects a new peril, bred by the very changes which he himself had done so much to accelerate. "The danger now is that of the entry of seven devils worse than the first—in the shape of rich engineers, and chemical traders, and "experts" (who have sold their souls for a good price), and who find it helps them to appear to the public as if they were scientific men." <sup>16</sup> Science had become so firmly entrenched that he trembled lest the spirit should be clogged by material rewards and external honors.

"Thickheads ought to recognize  
The Devil, that old stager, at his trick  
Of general utility, who leads  
Downward, perhaps, but fiddles all the way!"

To bring home the complete transformation through which Huxley lived, and did so much to mediate, take a final glance at the contrasts. When he was venting the first cries of lacteal consciousness, Liebig, then a very young *Ausserordentlich*, was pressing the government of

<sup>14</sup> *Life and Letters* (American ed.), vol. ii, p. 467.

<sup>15</sup> *Ibid.*, p. 336.

<sup>16</sup> *Ibid.*, p. 246.

little Hesse-Darmstadt to erect the pioneer laboratory at Giessen; in the year of his death, Rayleigh and Ramsay discovered argon! From no laboratories at all to the experimental refinements necessary for such analyses! Or, to take the biological field: when Huxley was still prattling in the nursery, his future guide Karl v. Baer, then in the fertile Koenigsberg period, published the *Epistola de Ovi Mammalium et Hominis Genesi* (1827); three years before Huxley's death, the capstone was placed on the huge literature of theory *pro* and *con* Evolution by Romanes' *Darwin and After Darwin*<sup>17</sup> (1892); nay, more, Huxley lived to see the beginning of the reaction against partizan generalities, and the return to investigation—destined to circle back upon v. Baer; for, Bateson's *Materials for the Study of Variation* came from the press in 1894! We take all this as matter of course, forgetful that Huxley protested against a state of mind and the practical stresses consequent upon it. Knowing both by their scandals rather than their fruits, he attacked them with irony, sometimes, be it confessed, with malice. Anxious to rid himself of sempiternal complications, he had recourse to an agnosticism more notable for rhetorical sharpness than for philosophical grasp; punctuating facts admirably, he was less successful in comprehension of events. Yet, if we make due allowance for the temporary perversities fought by him, we cannot fail to realize that the man overshadowed the job. As a result, he exerted pervasive *moral* influence, whereof we are the lucky legatees.

I have tried to picture the background of provocation. As we turn to the things of the personal mind, I imagine most of us would like to begin by applying to Huxley his own judgment of Carlyle. "Few men can have dissented more thoroughly than I from his way of looking at things; but I should not yield to the most devoted of his followers

<sup>17</sup> Open Court Publishing Company, Chicago.

in gratitude for the bracing wholesome influence of his writings."<sup>18</sup>

### III

Consideration for your comfort, let alone your patience, places strait limits upon my space. So, recall that Huxley was a typical Anglo-Saxon—a forward “yea-sayer,” when he sensed firm ground of empirical evidence, a dogmatic “nay-sayer,” when he came to those aspects of experience where, in the nature of the case, both ends lie out of sight. Indeed, one might allege that the defects of his qualities root just here. He takes with Puritan literalness the precept, “let your communication be Yea, yea; Nay, nay, and cannot abide shade. He is at home in high light or gross darkness. For these reasons, I feel justified in confining myself to two phases of his theoretical activity, promising *sans phrase* that they are closely connected. For, after all, his championship of “Darwinism” and his “philosophy” belong to a single whole.

From what we have seen above, it must be tolerably apparent that the “controversy over Darwin” characterizes a period difficult for us to realize. Nay, in a reference to Owen, written more than thirty years ago, Huxley himself declares: “the thing that strikes me most is, how he and I and all the things we fought about belong to antiquity.”<sup>19</sup> Yea, verily! How much the more, then, we must find ourselves in the plight of old Kaspar, when his grandchildren put him to the question about the battle of Blenheim:

<sup>18</sup> *Life and Letters*, vol. ii. p. 37.

<sup>19</sup> *Ibid.*, p. 395.

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"Now tell us what 'twas all about,"  
 Young Peterkin, he cries;  
 And little Wilhelmine looks up  
 With wonder-waiting eyes;  
 "Now tell us all about the war,  
 And what they fought each other for."

"It was the English," Kasper cried,  
 "Who put the French to rout;  
 But what they fought each other for,  
 I could not well make out;  
 But everybody said," quoth he,  
 "That 'twas a famous victory."

Accordingly, I inquire, What precisely was all the pother about? or, rather, Why should events have taken the course they did?

A man's capacity for reflection upon a problem depends to some extent upon such notions as may have been current previously. So it would be interesting to record the eclipse of emanational theories by the epigenesis form of "Evolution" which, note, really runs back to Aristotle's "perfecting," that is, "progressive" principle; or to relate the fortunes of naturalistic transformism in the struggle with supranaturalistic uniformitarianism in the seventeenth and eighteenth centuries; above all, to prove Darwin's debt to predecessors. But we must jump these "pregnant phases of the moon" to grapple with Darwin himself.

I have gone back to my *Origin* (I am the happy possessor of the first edition, which I used often to pull down from my father's shelves in the halcyon days when I played *Diener* to Romanes among the multitudinous jelly-fish of the Kyles of Bute). I still find Huxley correct; the *Origin of Species* "is one of the hardest books to understand thoroughly that I know of." And I recall his immediate re-

mark with no little amusement: "I suppose that is the reason why people like Romanes get so hopelessly wrong."<sup>20</sup> For, Romanes used to swear that Huxley and, as I recollect, much more emphatically Spencer, had "got the whole thing askew." What was a poor kid to do with the heavy-weight doctors scratching each others' eyes thus! Darwin is difficult, partly because he was a pioneer; partly because he could not but sense a hostile audience, expert and lay; partly because, with the probable exception of the *Fertilization of Orchids*, he is not readily "understood of the people"—as Huxley has it, "exposition was not Darwin's forte . . . but there is a marvellous dumb sagacity about him."<sup>21</sup>

Discounting present preoccupations, we must anticipate that matters peculiar to these early days will turn up, to give pause. In particular, it is difficult to reconcile Darwin's dispraise of Lamarck with his prevalent humility, with his usual courtesy to others. He speaks of "Lamarck nonsense" and, writing to Hooker in 1844, says: "with respect to books on this subject (descent of species from common stocks), I do not know of any systematical ones, except Lamarck's, which is veritable rubbish."<sup>22</sup> Nay, more, not content with relegating Lamarck to the same limbo as the author of the *Vestiges*—"his geology strikes me as bad, and his zoology far worse,"<sup>23</sup>—Darwin places in his mouth at least two views which he never held: "the will of organisms" as a cause of adaptation, and the "Vestigian doctrine of 'necessary progression,' that is, of progression independent of conditions." As I have hinted, not only is this far from Darwin's considerate attitude toward others—toward men unworthy to black Lamarck's boots—but it reappears in Huxley himself.<sup>24</sup> "To any

<sup>20</sup> *Life and Letters*, vol. ii, p. 204.

<sup>21</sup> *Life and Letters*, vol. ii, p. 203.

<sup>22</sup> *The Life and Letters of Charles Darwin*, vol. i, p. 390 (New York, 1896).

<sup>23</sup> *Ibid.*, p. 302.

<sup>24</sup> *Ibid.*, p. 542f.

biologist whose studies had carried him beyond mere species-mongering in 1850, one-half of Lamarck's arguments were obsolete, the other half erroneous, or defective." The significant point for our present purpose is—the explanation does not lie far afield. Darwin was obsessed at the moment by the *Allmacht* of Natural Selection, Huxley by the *method* that led to the discovery. As he verges upon the *Origin*, Darwin confesses: "I have come to the heterodox conclusion that there are no such things as independently created species—that species are only strongly defined varieties. . . . All my notions about *how* species change are derived from long continued study of the works of . . . agriculturalists and horticulturists; and I believe I see my way pretty clear on the means used by nature to change her species and *adapt* them."<sup>26</sup> The "Summary" of the famous fourth chapter of the first edition (p. 126f.) need not be quoted. Here is the "true cause" unveiled at long and last! It is fair to remind you that, keeping the open mind, Darwin made the *amende honorable* to Lamarck in a measure twenty-one years later, in the sixth edition: "This has been effected chiefly through the natural selection of numerous, successive, slight favorable variations; aided in an important manner by the inherited effect of the use and disuse of parts; and in an unimportant manner . . . by the direct action of external conditions, and by variations which seem to us in our ignorance to arise spontaneously" (p. 424). Here, Lamarck's and Buffon's "causes" are ranged alongside his own. I have cited these preliminaries because they supply the clue to Huxley's mental slant.

The clamant question framed by Lyell (1838) was, "the possibility of the introduction or origination of fresh species being a natural, in contradistinction to a miraculous process."<sup>28</sup> Put thus, the problem is amenable to the ordi-

<sup>26</sup> *The Life and Letters of Charles Darwin*, vol. i., p. 437f.

<sup>28</sup> *Life and Letters of Sir Charles Lyell*, vol. ii, p. 35 (London, 1881).

nary procedure of observation and classification, followed by inductive inference to a rule that holds uniformly within the group of phenomena under study, and by proof that the inference is consistent with the facts, inconsistent with another, especially a contrary, conclusion. All sorts of hypotheses had been set forth; Darwin was the first to hit upon the suggestion that (to use Huxley's language) "new species may result from the selective action of external conditions upon the variations from their specific type which individuals present—and which we call "spontaneous," because we are ignorant of their causation." He adds that this notion was "as wholly unknown to the historian of science as it was to biological specialists before 1858." And, referring to rival suggestions, more particularly "the ideal quasi-progressiveness of Agassiz," he points to the "want of any means of testing" their validity, invoking (unconsciously, I suppose) one of the logical laws for framing hypotheses—a hypothesis must be amenable to evidence *pro* or *con*. Finally, associating himself with contemporary investigators, who were all at sea, he points out that "the publication of the Darwin and Wallace papers in 1858, and still that more of the *Origin* in 1859, had the effect . . . of the flash of light, which to a man who has lost himself in a dark night, suddenly reveals a road which, whether it takes him straight home or not, certainly goes his way. . . . We wanted . . . to get hold of clear and definite conceptions which could be brought face to face with facts and have their validity tested. The *Origin* provided us with the working hypothesis we sought."<sup>27</sup>

Thus Huxley found "a hypothesis respecting the origin of known organic forms which assumed the operation of no causes but such as could be proved to be actually at

<sup>27</sup> The whole chapter, entitled "Professor Huxley on the Reception of the *Origin of Species*," in the *Life and Letters of Charles Darwin*, should be read.



work." <sup>28</sup> Fully persuaded in his own mind, and tense for intellectual virtue, he proceeded to act upon his motto—"the only freedom I care about is the freedom to do right," <sup>29</sup> and "played the part of something between maid-of-all-work and gladiator-general for Science," <sup>30</sup> because, "under the circumstances of the time, warfare" was his "business and duty." <sup>31</sup> All is plain sailing to this point, the imponderables being ponderable to some extent. For, the long and short of it is, that we are concerned with a particular induction from particular facts to the exclusion of all others; we are not concerned with anything in the nature of an ultimate theory of the universe. How came it about, then, that Huxley tended to treat the result as if it were a cosmic generalization?

## IV

To begin with, we must recall that no man can escape his age, whether in the matter of advantages (stressed as a rule), or of limitations (often left out of the reckoning). Now, Huxley began his official career as a lecturer on "Natural History," and this title happens to be tell-tale. Specialization in the biological and human sciences belonged to the future, particularly in England. On the other hand, the tradition of the *superficial* unity of the universe, as it might be termed, died hard. Leave German *Naturphilosophie* aside, to glance at the French development, more "scientific" because going back to Newton as one of its sources. Fontenelle's *Entretiens sur la Pluralité des Mondes* (1686), deriving from Descartes, marks the

<sup>28</sup> *Life and Letters*, vol. i, p. 182.

<sup>29</sup> *Ibid.*, p. 353.

<sup>30</sup> *Ibid.*, vol. ii, p. 173.

<sup>31</sup> *Ibid.*, p. 227.

onset of a movement. The later phase starts with Voltaire's *Elémens de la Philosophie de Newton* (1738), and gradually reaches flood in La Mettrie's *Histoire naturelle de l'Ame* (1745), and *L'Homme Machine* (1748), Buffon's *Théorie de la Terre* (1748, the first volume of his *Histoire Naturelle*), the *Encyclopédie* of Diderot and D'Alembert (1751f.), and "Mirabaud's" (Holbach) *Système de la Nature* (1770). All are philosophical works in the sense that they present systematic or universal theory; but none betray even a faint idea of the problem of metaphysics! Generalization supplies their common bond. They embody a medley, not a synthesis, of ideas seized at random from Newton (mechanical), Locke (epistemological), Shaftesbury (psychological), Linnaeus (biological), and Boerhaave (physiological). Of course, this kind of thing could not persist, but its presupposition, the notion of the unity of the universe, remained very much alive for a century. Whether the later developments ran in literary and quasi-philosophical directions, or to exact research, the concept of general "explanation" loomed in the background. Goethe summarized the situation with his customary insight: "If, after all, this book (Holbach's) did us any mischief, it was this, that we took a hearty dislike to all philosophy, and especially metaphysics, and remained in this dislike; while, on the other hand, we threw ourselves into living knowledge—experience, action, and poetising—with all the more liveliness and passion."<sup>32</sup> This was thoroughly characteristic; so was something else. Like Humboldt and J. R. Mayer and Darwin himself, Huxley spent his *Wanderjahre* exploring "Nature" at large, and the unity of "Nature" lent inspiration to them all. They were men of Aristotelian breadth, in the mould of Leibniz rather than of Newton.

<sup>32</sup> Quoted by F. A. Lange, *History of Materialism*, vol. ii, p. 150 (Eng. trans.). The fourth Section of Lange's First Book should be read for the French movement.

Once more, and reverting to Newton now, the old Natural Theology played its part also. Rooted in the doctrine of a "broken universe" (to which, recollect, Huxley reverted in his famous *Romanes Lecture*), it contemplated the existence of two orders. So far as possible, mathematico-physical principles are to be applied "where they apply." For the rest, the inbreak of a higher Power, evidenced by the operation of external Design, of a purpose spiritually conditioned and therefore fundamental to the physical, must be set up to "explain" universal adjustment. Hence, if "Natural Selection" render this "explanation" superfluous, then, obviously, the new "explanation" inherits the sweeping scope of the old. "Evolution" becomes a philosophical no less than the biological clue. Besides, this more or less unconscious *philosophical* substitution of "Evolution" for "Design" was rendered inevitable by the *animus* of the theological opposition. In a word, the universalism of the opposition forced a like scope upon Huxley in rebuttal. Nay, even more subtly, another influence, whereof he was quite unaware possibly, wafted him in the same direction, willy-nilly.

The desire for historical interpretation had become widespread—track a phenomenon to its origins, and nothing remains to be done; you "have it." Geology and the earlier studies of human culture (for example, Wolf's *Prolegomena ad Homerum*, 1795) gave a lead which biology followed inevitably as it broke loose from "Natural History." When Darwin wore down the partition between men and animals by obliterating the lines drawn between species, the fat of the historical method fell into the fire all round. To alter the figure, "Transformism" was the new Minerva, stepping forth a full-blown philosophy from the outset. In short, a single and universal conclusion had been fated to overleap the complexities of detail, as biologists were to recognize increasingly after 1889. Accord-

ingly, Huxley felt no urge to the "labor of the notion"; and so the sound of his trumpet is uncertain. "The safety of morality lies neither with the adoption of this nor that philosophical speculation, or this or that philosophical creed, but in a real and living belief in that order of nature which sends social disorganization upon the track of immorality." He seems unaware that the words "real and living belief" in an "order of nature" are precisely "the adoption" of a definite "philosophical speculation and creed." For this reason, his philosophy could not but be superficial, or, as my colleague Doctor Sellars well says, naive.<sup>23</sup> I shall conclude with a few words on this aspect of the subject, premising, however, that the Agnosticism must be tackled at another time.

Till Kant, modern European thought proceeded along two lines, one peculiar mainly to the Continent, the other to England. The former, dominated by ontological interest, laid stress upon the primacy of "clear and distinct ideas," which thought is able to grasp by its own inherent capacity. As Descartes contends, these ideas proceed *not* from external objects, but only from my capacity to think. If so, then evident propositions cannot be drawn from sensuous sources, even if particular phenomena suggest self-evident principles. For the Cartesians, accordingly, sense perception becomes an inferior grade of thinking, inferior because "confused." Sure foundations for experience must be sought in an *a priori* knowledge after the mathematical type. Dominated by the psychological and epistemological interest, the British temper denied "an innate cognoscitive power of raising intelligible conceptions of things from within" the mind itself, and asserted that men "may attain all the knowledge they have . . . without any such original notions or principles." Hence, naturally, a drift toward the doctrine that sense-presentation furnishes

<sup>23</sup> *Evolutionary Naturalism*, p. viii, Open Court Pub. Co., Chicago.

the mind with specific information about outer things. Sensation coerces, as Locke hints broadly. "I ask anyone whether he is not invincibly conscious to himself of a different perception, when he looks on the sun by day, and thinks on it by night; when he actually tastes wormwood or smells a rose, or only thinks on that savour and odour." Lesser and hastier empiricists, intent upon the obvious, as lesser and hastier folk always are, inevitably ran away with the harrows, and came to hold that thought is nothing more than a dim or dull revival of sensuous states. Here, then, we have the origin of the contrast between "metempiric" and "empirical" knowledge—mere zest for "abstraction" set over against contact with sensible "concrete fact." Hence, experience was readily divisible into the "merely mechanical" and the "purely mental," with easy oblivion of the double reference of sensation; to a subjective state and to an object of apprehension. This was the presupposition determining such philosophy as Huxley had.

If there be two series, quite out of any relation except temporal occurrence, then knowledge of both lies beyond the bounds of practical politics. In this dilemma, incipient physiology rendered it more or less unavoidable that sensuous states should be taken as "representations" of the actual course of the external order. An otiose agnosticism would then become the final result; otiose because, after all, the mechanical scheme continues to dictate the conditions. Locke may asseverate, "general certainty is never to be found but in our ideas," and Huxley may agree in principle. Despite theory, atoms are still the "foundation stones" of the universe, as he and Tyndall have it. Huxley fell into this trap with many another layman. He had not fathomed Hume's reduction of empiricism to scepticism. As has been pointed out repeatedly, his brilliant *tour de force* in the "English Men of Letters" Series (1879) is

no more than a "relatively uncritical exposition."<sup>34</sup> Any writer of equal literary equipment might have done as good a job, save for certain psychological excursions! To make matters worse, Huxley had not grappled with Kant, much less with Hegel. Paradoxically enough, he would have fared better as a philosopher had he gone over to the extreme Left of materialism, from which, as he persistently said, countering widespread misrepresentation, he dissented. His epoch would not permit this. He lived at the close of a period when gravitation, magnetism, electricity, and all other "forces" were reckoned "properties of matter." Although he was shaken, so to speak, by the rising notion of "energy," his traffic with fundamental ideas did not go deep enough to enable him to realize whither such a concept as Maxwell's, of the correlation between light and electricity, was actually leading.

In effect, then, Huxley dismissed the materialistic theory, and proceeded to accept it pragmatically. "The materialistic terminology is in every way to be preferred. For it connects thought with other phenomena of the universe." The "primacy of the physical" is convenient practically. I am free to say that Huxley never rose above amateurishness of this type. For, when he tells us that the "plain duty of each is to try to make the little corner he can influence . . . somewhat less ignorant," he adds; "our volition counts for something in the course of events." Very good. But, what is "our volition"? "The physical state of which our volition is the expression," as he amends, in the same context, twenty-four years later. It were superfluous to convict him of inconsistency; he merely did not see the point, being out of his field of competence. And there can be little doubt that *naïveté* of this kind was rendered inevitable by the "practical" tendency of the British schools with which he had affiliations. Utilitarianism, moralism, and

<sup>34</sup> Cf. M. W. Calkins, *The Persistent Problems of Philosophy*, p. 501.

the supposititious unanswerableness of Hamilton and Mansel may be saddled with his myopia.

Seeing that Huxley was moved by wish and will much more than by intellect in his attitude toward the interpretation and conduct of life, we may well afford to disregard his philosophy, treating it as a side-issue. So we dismiss it, to commemorate a pioneer investigator of manifest gifts, backed by magnificent courage at a juncture when courage counted; an expositor seldom rivaled in the combination of skill, eloquence, and lucidity with first-hand knowledge; a writer of persuasive charm; a controversialist contemptuous of petty issues and pettier dodges; a citizen who accounted loyalty a spiritual experience; above all and, perhaps, explaining all, a great soul who scorned iniquity and cleaved unto uprightness, utterly without reference to selfish ends. We dwell upon him here and now, in this far land, because each of us is a personal debtor to his fealty and example.

R. M. WENLEY.

UNIVERSITY OF MICHIGAN.



## MEANING IN THE CASE OF MATHEMATICAL POSTULATES

THE notion of meaning furnishes one of the most difficult, and at the same time one of the most important, problems of philosophy and psychology. The fact that we conceive meanings stands as a perpetual challenge to and refutation of every grossly materialistic system of philosophy or psychology. Up to the present there is no intimation of any way by which meanings may be gotten by, or explained through the use of, matter and motion and energy; nor is there any reason to believe that success in this direction shall ever become possible. Meaning is something contributed by the mind and is not in the physical matter-energy situation which is involved between a thinker and his environment. It is so elusive in its nature that we can not fail to prize any aid to its understanding or any clarification as to the nature of the problem in any of its aspects. Its role in mathematical postulates and the theories arising from them is such as to enable us to see the whole problem of meaning in a light or under a color which helps us to sense a feature of its character that we might otherwise miss entirely. It is my present purpose to exhibit this notion with the illumination and the coloring given to it by postulational systems in mathematics and their consequent theories.

## I

Perhaps we may partially state in advance the nature of the problems of meaning which are set by the development of a doctrinal function based on a system of mathematical postulates. Here we always have to deal with one or more "undefined" terms and "undefined" relations. It is clear that these, though not explicitly defined, can not be entirely devoid of meaning; for, in that case, we should have no grip upon them to enable us to deal with them in reasoning. But they need not have unique specific concrete meanings. In fact, in the more important cases, the number of concrete meanings or interpretations which are consistent with the postulates is usually infinite. All the arguments deduced in the development of the doctrinal function and all the results obtained must be valid for all these interpretations, even though the person developing them knows only a part or none of these interpretations. The "undefined" terms and relations are variables and must be allowed to have any interpretations whatever which are consistent with the postulates. It is clear, therefore, that the argument proceeds by means of the forms of these basic postulates or by means of the properties or qualities assigned by them to the undefined entities. The meaning of these entities is variable; and the range of variation may be infinite so that it can not possibly be contemplated in its full variety during the development of the argument. In fact, the argument can proceed, a part of the way at least, while the person developing it is totally ignorant of the interpretations. It is clear that such a procedure must raise delicate questions concerning the meaning of what is being done. It gives, indeed, a new

color or a new characteristic to the question of the meaning of meaning.

There is a tendency, on the part of some thinkers, to assert that the nature of reasoning is the same wherever it occurs, whether in mathematics or in natural science or in the less precise processes which are closer to the activities of everyday life. With this judgment we have no present purpose to disagree; it is largely a matter of the definition of the nature of reasoning. But, if the nature of reasoning is to be treated as unique, that nature must be so broadly described as to allow a place for the sort of reasoning that is involved in developing a doctrinal function. We shall see that the nature of reasoning has been described in such a way as to be inadequate to cover the processes of which we are to give an account in the present paper. Either a more comprehensive description of this nature must be attained to or it must be admitted that we have not yet found a way to describe reasoning so as to comprehend certain of the processes which are familiar to the mathematician and are widely used by him.

There is no purpose on our part to assert that the same processes are not employed outside of mathematics. Whether one holds this judgment or not will depend primarily on what he considers to be the sameness of nature of two or more pieces of reasoning. That other exact sciences, in their progressive development, tend towards the employment of just the sort of reasoning used in mathematics can not be doubted; but we believe that certain characteristics of reasoning stand out so much more clearly in mathematics than elsewhere that it is better to consider them first in relation to that subject. And this we shall do in the present paper.

The processes to be described are constantly present in a large part of mathematics. Since this article is written principally for those who are not conversant with the de-

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tails of mathematical development, it seems desirable to set before the reader an example of a system of mathematical postulates and the doctrinal function based on them. For this purpose we have chosen a very simple system, one which is subsumed under the more general system on which is based the centrally important theory of finite groups. We place before the reader all that he will need for the understanding of the particular system employed, provided that he is willing to think consecutively and abstractly—an act which is not easy for all persons, or indeed for all exact thinkers.

## II

To achieve our present purpose it is necessary to set forth briefly a particular system of postulates and to analyze the general notion of doctrinal function which this system will enable us to make clear. We begin by defining certain preliminary terms of a rather abstract character. We are concerned with mathematical objects, that is, objects so clearly conceived or defined or *so fully characterized as to relevant properties* as to be subject to the precise analysis required in mathematics. We shall treat certain of these objects in sets or classes each of which contains a finite number of the objects. The objects in the sets which we shall now need to consider will present themselves to our thought in connection with certain rules of combination by means of which a new object is obtained from two given objects when combined in a certain order. The new object, in general, may or may not belong to the given set; but, in the sets which we are to treat in detail this new object will belong to the set. In a particular set

there may be just one rule of combination, or there may be more. The numbers of ordinary algebra, considered as objects in the sense now intended, are subject to the two fundamental rules of addition and multiplication. The process by which two points determine a line in geometry affords a rule of combination of these objects so as to produce a new object of a different sort. The combination of two displacements produces a third displacement. A set of objects with the associated rule or rules of combination, is called a system, or, more explicitly, a mathematical system.

Certain important and frequently occurring systems are called groups. When the number of elements in a group is finite it is called a finite group. For the sake of simplicity we shall here take a very simple class of groups. The objects occurring in a finite group are called its elements, and their number is called the order of the group. These elements are subject to a single rule of combination. If the elements employed are of such a nature that they also have one or more other rules of combination the latter will be ignored, or at least will not be employed, in treating the elements in their relation to the group in consideration. We shall now give the defining postulates for the particular class of groups which we are to consider.

Let  $G$  be a system consisting of a set of elements and one rule of combination for uniting any pair of them in a given order. The elements and the rule of combination for uniting them are undefined except for the characteristics or qualities or properties implied by the postulates presently to be stated. If  $a$  and  $b$  are two elements of  $G$  we shall denote by  $ab$  the element resulting from combining  $a$  with  $b$  in the order written; similarly we denote by  $ba$  the element resulting from combining  $b$  with  $a$  in this latter order. (In the general theory of groups the elements  $ab$  and  $ba$  need not be equal; in the particular class

of groups here treated we shall prove that they are equal.) If two symbols or groups of symbols denote the same object we shall express that fact by writing one of them equal to the other, using for this purpose the usual sign of equality. Let the "undefined" elements in  $G$  and the "undefined" rule of combination of them in pairs in a given order be subject to the following five conditions or postulates:

1. If  $a$  and  $b$  are any elements of  $G$ , whether the same or different,  $ab$  is also an element of  $G$  and is uniquely determined by the rule of combination. (This will be called the "group property" postulate, since it sets forth the most important property of a group.)

2. If  $a, b, c$  are any elements of  $G$ , then  $(ab)c$  is the same element as  $a(bc)$ , the parentheses indicating that the enclosed elements are first to be combined in the order written and that the result is to be combined with the other element in the order indicated by the order of symbols. (This means that the associative law holds for the given rule of combination as applied to the given set of elements.)

3. The set of elements in  $G$  contains a single element  $i$ , called the identity, such that for every element  $a$  of the set we have the relations  $ai = ia = a$ .

4. If  $a$  is an element of the set there is also an element  $a'$  such that  $aa' = a'a = i$ . Moreover, there is only one such element  $a'$ . (The element  $a'$  is called the inverse of  $a$ ; it is clear that  $a$  is likewise the inverse of  $a'$ .)

5. The number  $n$  of elements in  $G$  is a prime number.

If in place of 5 we had put the less stringent requirement that the number of elements in  $G$  is finite we would have had the general definition of finite group. We are therefore dealing with a special case of the important subject of finite groups; in fact, the case that we have taken is exceedingly more simple than the general case, but it

serves equally well our present purpose and will be much easier for the non-mathematical reader.

We shall not undertake to treat in detail the measure of independence of these postulates, independence in the sense that no one of them is deducible from the others by logical processes alone. In some connections that problem is of great importance, but for our present purpose (in the study of the nature of meaning) it is negligible. If we were engaged in the analysis of the intimate logical connections of the doctrinal function we are developing this would be a question of central importance and would require a careful study. But the results of that analysis would contribute little to the questions we propose to raise concerning the nature of meaning and of the processes of reasoning employed in developing a doctrinal function.

But it is important to study the question of consistency. In order to be sure that these postulates are consistent in the sense that no two logical consequences of them can possibly be mutually contradictory we shall exhibit a system of elements satisfying them. In fact, in order to meet certain other needs arising later we shall exhibit three different systems satisfying them. Let

$$1, w_1, w_2, \dots, w_{n-1}$$

be the  $n$   $n$ th roots of unity and let the law of combination among them be that of ordinary multiplication. Since the product of any two  $n$ th roots of unity is also an  $n$ th root of unity, it follows that postulate 1 is satisfied. Postulate 2 is satisfied in view of the associative law of multiplication. The element 1 is the identity and has the properties required in postulate 3. As the inverse of any given element we have its reciprocal, this assuring us that postulate 4 is satisfied. Postulate 5 is satisfied if we restrict  $n$  to be a prime number.



As a second example, let us consider certain rotations of a plane about a point  $P$  in the plane. Let  $\omega$  be an angle such that  $n\omega = 360^\circ$ . Then let the elements of  $G$  consist of the rotations about  $P$  of angular measure

$$\omega, 2\omega, 3\omega, \dots, n\omega.$$

We shall call  $(n+1)\omega$  the same rotation as  $\omega$ , since it leaves the plane in the same final position. Let the rule of combination among the rotations be that of addition, so that the result of combining two rotations is a single rotation equal to the sum of the two given rotations. The identity is the rotation  $n\omega$ . The inverse of the identity is the identity itself. The inverse of  $k\omega$ , where  $k$  is different from  $n$ , is  $(n-k)\omega$ . It is now easy to see that this system satisfies all the postulates set up for the group  $G$ , provided that  $n$  is a prime number.

As a third example, consider the following: Let the  $k$ th element in  $G$  be the operation of replacing the sequence of symbols

$$a_1, a_2, a_3, \dots, a_n$$

in order by the sequence

$$a_k, a_{k+1}, \dots, a_n, a_1, a_2, \dots, a_{k-1}.$$

If such an element is formed for each value  $k$  of the set  $1, 2, 3, \dots, n$ , we shall have  $n$  elements to form the system  $G$ . For  $k=1$  we have the identity. If in one operation  $a_i$  is changed to  $a_j$ , and if in another  $a_j$  is changed to  $a_i$ , then the result of combining these two operations in the order named will change  $a_i$  into  $a_i$ ; this will serve to define the rule of combination of elements of  $G$ . When the elements and the rule of combination are so defined and  $n$  is a prime number, the system satisfies the five postulates, as the reader may easily verify.

Each of these examples affords what may be called an interpretation of the system of postulates. The three interpretations involve elements of widely different sorts and rules of combination with little that is concrete in common. Any one of the examples is sufficient to show that the set of postulates is consistent. For we are certain that every logical consequence of the postulates is a statement which is true of the objects in any one of the sets selected for examination. But no existent thing can have two mutually contradictory properties. Hence no two logical consequences of the postulates can be mutually contradictory. The several examples taken together are sufficient to show that a variety of interpretations is possible. It would be easy to exhibit many other systems  $G$  satisfying the same postulates. In fact the number of them is unlimited. In deriving the consequences of the postulates, that is, in obtaining the theorems which flow from them, one must use only those statements which afford true propositions for every possible interpretation; and he must do this without knowing all these possible interpretations. He must therefore in some way work with the "undefined" elements and "undefined" rule of combination by means of those properties alone of them which are stated in or are implied by the given postulates; or he must proceed by means of the forms of the propositions without having in mind a precise realization of their content of meaning. It is this dealing with an abstract or variable situation in such a way as to be universally valid, that is, valid for all possible interpretations, that raises the problem of meaning in the characteristic form in which it appears in the study of the consequences of mathematical postulates. In order to illustrate the nature of the thought processes involved, we shall now derive some of the principal theorems implied by the foregoing postulates.

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## III

In the proof of the theorems which are now to be developed we shall have to ask the reader's indulgence for the presentation of the necessary matters of detail. They are necessary if he is to have in mind an example of the type of reasoning which we are analyzing.

Let  $a$ , or  $a^1$ , denote any element of  $G$ . Let  $a^2$  denote the element  $aa$  obtained by combining  $a$  with  $a$ . Similarly let  $a^3$  denote the element  $a^2a$ , this being equal to the element  $aa^2$  in view of postulate 2 which says in particular that  $a(aa)$  is the same as  $(aa)a$ . In general, when  $k$  is greater than unity, let  $a^k$  denote  $a^{k-1}a$ . We shall call  $a^s$  the  $s$ th power of  $a$ ,  $s$  being an integer. By applying postulate 2 repeatedly, we have successively the following relations:

$$\begin{aligned} a^k &= a^{k-1}a = (a^{k-2}a)a = a^{k-2}(aa) = a^{k-2}a^2 \\ &= (a^{k-3}a)a^2 = a^{k-3}(aa^2) = a^{k-3}(a^3) = a^{k-3}a^3. \end{aligned}$$

By continuing this process we may show that

$$a^k = a^{k-l}a^l$$

where  $l$  is any positive integer less than  $k$ . If we use  $a^*$  as a symbol for the identity element  $i$  we see from the foregoing result that

$$a^s a^t = a^{s+t}$$

if  $s$  and  $t$  are non-negative integers.

Let us denote by  $a^{-1}$  the inverse of the element  $a$ , and by  $a^{-k}$  the  $k$ th power of this inverse  $a^{-1}$ . Then it is easy to see that the "law of exponents"

$$a^s a^t = a^{s+t}$$

holds for all integral (positive or negative or zero) values

of  $s$  and  $t$ . Thus the "rule of combination" has one of the important fundamental properties of multiplication.

Now consider the infinitude of symbols for elements

$$a^0, a^1, a^2, a^3, a^4, \dots$$

Each of these symbols denotes an element of the group  $G$ , as we see from postulate 1. Since the number of elements in  $G$  is finite it follows that there must be certain equalities of elements among this infinite set. Let  $a^k$  and  $a^m$  be two of them that are equal,  $m$  being greater than  $k$ . Then we have

$$a^m = a^k.$$

On combining each member of this equation with  $a^{-k}$  we have

$$a^m a^{-k} = a^k a^{-k}.$$

The second member of this equation is equal to  $a^0$  and hence denotes the identity element  $i$ . Therefore

$$a^{m-k} = i.$$

Therefore some positive integral power of  $a$  is equal to the identity. Let  $d$  be the least positive integral exponent for which

$$a^d = i.$$

It is now easy to see that any given power of  $a$  is equal to some one of the elements

$$a^0, a^1, a^2, \dots, a^{d-1}.$$

We shall now prove that  $d=n$  when  $a$  is not itself the identity element.

Let  $a$  now be an element different from the identity and let us consider the set of  $d$  elements from the set  $G$ , namely:

$$(1) \quad a^0, a^1, a^2, \dots, a^{d-1}.$$

No two of these are equal; for, if  $a^s$  and  $a^t$  are two of these and  $t$  is greater than  $s$  and if  $a^t = a^s$  we have  $a^t a^{-s} = a^s a^{-s}$  or  $a^{t-s} = i$  contrary to the hypothesis that  $d$  is the least positive exponent such that  $a^d = i$ . Since no two of the  $d$  elements in (1) are equal and since each of them is in  $G$  it follows that the number  $d$  of them is not greater than the number  $n$  of elements in  $G$ .

If (1) contains all the elements of  $G$  then  $d = n$ , and the result to be established is granted.

If we assume that (1) does not contain all the elements of  $G$ , we shall be led to a contradiction, as we shall now show. In this case let  $b$  be an element of  $G$  which is not contained in (1). Then all the elements

$$(2) \quad a^0 b, a^1 b, a^2 b, \dots, a^{d-1} b$$

are contained in  $G$ , as one sees by the use of postulate 1. We shall show that the  $d$  elements in (2) are different from each other and also different from the elements in (1). In the first place if  $a^t b = a^s b$  and  $t$  is greater than  $s$  we have  $a^t b b^{-1} = a^s b b^{-1}$ , or  $a^t i = a^s i$ , or  $a^t = a^s$ ; and this contradicts the known fact that the elements in (1) are all distinct. In the second place, if  $a^u = a^v b$  we have  $a^{-v} a^u = a^{-v} a^v b$  or  $a^{u-v} = i b$ , or  $a^{u-v} = b$ . Since  $a^{u-v}$  is obviously contained in (1) it follows from this that  $b$  is contained in (1), and this is contrary to hypothesis. Therefore no element in (2) is equal to an element in (1).

From these results we see that the sets (1) and (2) contain  $2d$  distinct elements of  $G$ . If these are not all the elements of  $G$ , let  $c$  be an element of  $G$  which is not in (1) or in (2). Then the elements

$$(3) \quad a^0 c, a^1 c, a^2 c, \dots, a^{d-1} c$$

are all in  $G$ , as one sees from postulate 1. That they are different from each other and from the elements in (1) is proved by precisely the same arguments as those employed

in treating the set (2). That they are different from the elements in (2) may be shown in the following manner: If  $a'b=a'c$  we have  $a^{-1}a'b=a^{-1}a'c$ , or  $a^{-1}b=c$ . But  $a^{-1}b$  is obviously in (2); and hence  $c$  is in (2), contrary to hypothesis.

From these results it follows that the sets (1) and (2) and (3) contain  $3d$  distinct elements of  $G$ . It is obvious that the process may be continued until all of the elements of  $G$  are included in subsets of  $d$  elements each like the subsets (1) and (2) and (3). Let the number of the subsets be  $l$ . Then the number of elements contained in all of them is  $ld$ . Hence  $ld=n$ ; that is,  $n$  is divisible by  $l$  and  $d$ , neither of these numbers being unity. But this is contrary to postulate 5 which asserts that  $n$  is a prime number. The hypothesis which led to this contradiction must be discarded. Hence, we conclude that the elements (1) are all of the elements of  $G$  and therefore that  $d=n$ .

We have thus established the following general theorem:

If  $a$  is an element of  $G$  which is different from the identity then each element of  $G$  is given once and just once in the set

$$(4) \quad a^0, a^1, a^2, a^3, \dots, a^{n-1}.$$

Moreover, we have  $a^n=a^0=i$ .

We may then say that the set in  $G$  is generated by taking the powers of any single element  $a$  of  $G$ , provided that  $a$  is different from the identity. This gives us a comprehensive understanding of the abstract nature of the groups defined by our set of postulates. There is one such abstract group for each prime number  $n$ .

## IV

We have given the foregoing system of postulates and the arguments connected with them not for their own sake but for the purpose of setting before the reader a simple example of a set of mathematical postulates by means of which example we shall be able to make clear the nature of the main considerations which we wish to raise concerning the problem of meaning. The example is not sufficiently comprehensive to bear on every aspect of our investigation; but it will serve as the point of departure for our main arguments. With it before us we may now proceed to the main purpose of the paper.

Let us consider the nature of the statements made in the postulates. They have the forms of propositions; but they lack one thing of being propositions, in the strict sense of the term. A proposition, strictly so called, is a statement of fact or truth or falsehood about the entities to which the proposition refers. It is either true or false. This requires that those entities shall be defined; they have a definite constant meaning. They are not variables. Thus when I say, Socrates is a man, the terms which I employ are constant in their meaning and the statement is a proposition. But when I say,  $x$  is a man, I use a statement which is not a proposition. As it stands, the statement is neither true nor false. It contains an unknown element  $x$ , an element which can be assigned in an unlimited number of ways, an element then which is a variable in the sense that it is capable of various interpretations. The statement,  $x$  is a man, has the form of a proposition but is not strictly a proposition. Using the terminology of Bertrand Russell, we may call it a propositional function. A propositional function, then, is a statement having the form of



a proposition but differing from a proposition in the fact that it contains one or more variable elements. It must be such as to become a proposition when the variable elements are assigned constant values.

A propositional function is neither true nor false. It has the form of a proposition and may become either true or false or without meaning when the variable elements are assigned constant meanings. Thus the propositional function " $x$  is a man" becomes the true proposition "Socrates is a man" when the variable  $x$  is given the constant value "Socrates." When  $x$  is given the constant value "a dog" it becomes the false proposition "a dog is a man." The statement,  $x$  is a positive integer less than 5, becomes a true proposition if  $x$  is given any one of the values 1, 2, 3, 4; it becomes a false proposition if  $x$  is replaced by any other positive integer; if  $x$  is replaced by "the class of all positive integers less than 5" it becomes the nonsense "the class of all positive integers less than 5 is a positive integer less than 5."

To see clearly what the term "propositional function" indicates is a first step toward understanding the meaning of a system of postulates. Consider the first postulate in the set which we have treated in the foregoing pages. It refers to the variable system  $G$  and the variable elements  $a$  and  $b$  belonging to  $G$  and the variable rule of combination for uniting the elements of  $G$ . It is therefore not a proposition. So long as the variables in it remain variables the statement is neither true nor false. It has the form of a proposition without being one. We have explained three different ways in which these variables may be determined so that the statement shall become a true proposition. But as it stands, it is a propositional function rather than a proposition. Similar remarks could be made about each of the other postulates in the set. Each of them is a propositional function.

In general the variables in a propositional function may be determined in many ways so that the function becomes a true proposition. The sets of constants over which the variables in a given propositional function may range subject to the condition that every resulting proposition shall be true may be called the truth-range of the variables in the propositional function.

In the set of postulates which we have given we have five propositional functions. For each of them the variables involved have a certain truth-range. Since we have five postulates we may consider five truth-ranges, one for each of them. These five truth-ranges have a certain part in common as is apparent from the fact that we have shown (in three ways) how the variables may be assigned constant meanings so that all the postulates become true propositions. The entire common part of these five truth-ranges we may call the truth-range of the set of postulates.

If we take from this truth-range of a set of postulates any set of constant values for the variables in it we have what may be called an interpretation of the set of postulates. Three such interpretations we have already exhibited. In the next section we shall treat these interpretations with more fulness.

Let us now consider further the body of statements consisting of the postulates and the consequences which flow from them by logical processes. That such consequences may be derived we have already shown by exhibiting some of them together with their derivation. From the given statements we have extracted others. By aid of the general theorem with which we ended our analysis we have a much clearer insight into what is implied by the postulates than we are able to obtain by a direct inspection of the postulates themselves unaided by the chain of systematic deductions. The resulting statements have necessarily no further implications than those which are in-

volved in the postulates themselves; but the implicit knowledge of the system *G*, involved in the postulates, is much more explicit in our concluding general theorem than it was in the postulates to begin with. Logically we have nothing new; but, as regards our insight into the system, we have made distinct progress. We could also get a much fuller knowledge of it by proceeding still further with the deductions.

Now let us inquire as to the nature of the totality of statements contained in the postulates and their logical consequences. Since all the original statements involve variables it is natural to expect variables in some at least of the consequences flowing from them. That this expectation is actually realized is shown by the deductions already exhibited. What is the form of the body of statements consisting of the postulates and their logical consequences? It is evidently that of a doctrine, the latter being defined as a set of true propositions and the propositions implied by them. But this body of postulates and their consequences is not a doctrine because some at least of the statements contained in it are propositional functions rather than propositions. What then may we call this body? A most appropriate name is suggested by Keyser in his *Mathematical Philosophy* (Dutton, 1922); he calls it a doctrinal function.

Let us now consider what change in nature is effected in the doctrinal function when we assign to its variables a particular set of constant values. In the case of the postulate system which we are treating let us assign to the variables one of the interpretations which we have already indicated. Then each postulate is transformed into a true proposition and likewise each of their logical consequences is transformed into a true proposition; moreover the latter propositions are logical consequences of the former. Thus we see that a doctrinal function is converted into a

true doctrine by assigning to the variables in it any set of constants belonging to the truth-range of the basic set of postulates. The doctrinal function is not a doctrine but a form of doctrines; it is transformed into a doctrine by every assignment of constant values to the variables in such a way that the postulates become true propositions.

It is to be observed that the development of a doctrinal function from its postulates proceeds in terms of the variables in the postulates. The logical steps in the argument are taken without reference to the interpretation. The development of a doctrinal function is not a simultaneous development of all the doctrines into which it may be transformed though it implies the validity of each resulting doctrine as soon as it is conceived. A doctrinal function can be (at least partially) developed as such by one who knows only a part, and even by one who knows none, of its interpretations.

## V

In order to make it abundantly clear that a doctrinal function may be developed and understood by one who is ignorant of some at least of its important interpretations we shall set forth and discuss another interpretation of the system of postulates already given.

Let  $p$  and  $s$  be two odd prime numbers such that

$$p = 2s + 1.$$

That such pairs exist is seen by examining the following as possible values for  $s$ : 3, 5, 11, 23, 29, 41, 83, 89. We restrict  $n$  in postulate 5 to be a prime  $s$  such that  $2s + 1$  is

a prime  $p$ . We define the  $s$  elements of  $G$  to be

$$(5) \quad 1^2, 2^2, 3^2, \dots, s^2.$$

Then postulate 5 is satisfied when  $n=s$ . For the rule of combination we take the following: The combination of  $\alpha^2$  with  $\beta^2$  is to give  $\gamma^2$  where  $\gamma$  is the positive or negative number of the set  $\pm 1, \pm 2, \pm 3, \dots, \pm s$ , obtained as a remainder on dividing  $\alpha\beta$  by  $p$ , it being clear that such a remainder always exists uniquely. Then  $\gamma^2$  is that number of the set (5) such that  $\alpha^2\beta^2 - \gamma^2$  is divisible by  $p$ . It is clear that only one such number  $\gamma^2$  exists. Then postulate 1 is satisfied. It is almost immediately obvious that postulate 2 is satisfied. For the identity element  $i$  we take  $1^2$ , whence it follows that postulate 3 is satisfied. In order to prove that postulate 4 is satisfied let us consider the set of numbers

$$(6) \quad 1, 2, 3, \dots, 2s$$

and the corresponding set

$$(7) \quad \alpha, 2\alpha, 3\alpha, \dots, 2s\alpha,$$

where  $\alpha$  is any number of the set  $1, 2, 3, \dots, s$ . On dividing the successive numbers in (7) by  $p$  we get certain positive remainders less than  $p$ ; we denote these in order by

$$(8) \quad a_1, a_2, a_3, \dots, a_{2s}.$$

It is easy to see that no two of the numbers (8) are equal. Moreover, each one of them is a number of the set (6), and (8) and (6) have  $2s$  elements each. Hence some one of the number (8), say  $a_k$ , must be the number 1 of the set (6). If  $k$  is not greater than  $s$  we take  $\alpha'$  to be  $k$ ; if  $k$  is greater than  $s$  we take  $\alpha'$  to be  $p-k$ . Then, on dividing  $\alpha\alpha'$  by  $p$  we have a remainder 1 in the first case and a remainder  $-1$  in the second case. Hence on dividing  $(\alpha\alpha')^2$

by  $p$ , we have the remainder  $1^2$ . Therefore  $(\alpha')^2$  is the inverse of the element  $\alpha^2$  in (5). Therefore an inverse element exists for every element in (5). It is easy to see that it is unique. Hence postulate 4 is satisfied. Therefore the set of elements (5) with the defined rule of combination affords an interpretation of our system of postulates for the case of every prime  $n$  such that  $2n+1$  is also a prime.

Probably there will be no reader of this paper who will have had this last interpretation in mind at the time when he read the first four sections of the paper. Certainly the author did not have it in mind at the time when he wrote the earlier part of the paper. He had never observed the precise facts involved in this interpretation until he noticed them in seeking to find an interpretation of the foregoing postulates to serve the purposes of the remainder of this paper. The development actually given for the doctrinal function in consideration was worked out, and the argument may be fully understood without any reference to the interpretation just given.

And yet the reader who has followed the argument closely will be convinced that the conclusions reached in the study of the doctrinal function will all be valid when applied to this new interpretation—notwithstanding the fact that its existence had not been observed at the time when the (partial) development of the function as given in the foregoing pages was carried out. Let us see what some of these conclusions amount to under the present interpretation.

In the general theory we saw that the  $n$ th power of a non-identity element of  $G$  is the lowest power of it which is equal to the identity. For the interpretation now in consideration this means that the lowest even power of one of the numbers 2, 3, 4, . . . ,  $s$  which yields unity as a remainder after division by  $2s+1$  is the 2st power. This

is a significant fact about prime numbers of the form  $2s+1$  where  $s$  itself is a prime number. We are now certain of the truth of the conclusion reached without an independent examination of this case, since we know that the numbers (5), with the named rule of combination, furnish an interpretation of our set of postulates—and this is true, notwithstanding the fact that this interpretation was not in mind at all at the time when the doctrinal function was developed.

To illustrate the particular conclusion given in the preceding paragraph let us take  $s=11$ , whence  $2s+1=23$ . Let us consider the particular number  $3^2$  of the set (5). The even powers of 3 up to the twenty-second, namely,

$$3^2, 3^4, 3^6, 3^8, 3^{10}, 3^{12}, 3^{14}, 3^{16}, 3^{18}, 3^{20}, 3^{22}$$

yield in order the following numbers as the remainders on division by 23:

$$(9) \quad 9, 12, 16, 6, 8, 3, 4, 13, 2, 18, 1.$$

Thus the first of these even powers to give the remainder 1 is the 22nd, in accordance with the general conclusion already reached.

If we should take another number of the set (5), say  $2^2$ , and form (with  $s=11$ ) the set of remainders analogous to set (9), then we should get merely the numbers of that set in another order, as we know in advance from the general theorem already proved on the basis of the original postulates. Still other properties of sets (5) follow immediately from the general theory. If the given doctrinal function had been developed in great detail we would be able to conclude (without further argument) to many number-theoretic properties.

It must now be clear that a doctrinal function can be developed independently of its interpretations and that it can *afterwards* be made to yield significant doctrines.



Important doctrinal functions have an infinitude of interpretations and these may be of the widest variety as to concrete properties. For the doctrinal function which we are considering we have now given four distinct interpretations of different characters, three of them applying for every prime number  $n$  and the fourth one applying for those prime numbers  $n$  which are such that  $2n+1$  is a prime. Of these interpretations, one is algebraic, referring to the algebraic roots of unity; the second is mechanical, referring to rotations; the third refers to permutations of letters; and the fourth refers to positive integers. An unlimited number of additional interpretations could be defined, and some of these would have concrete characteristics different from those of the four already named. But we have given a sufficient variety of interpretations to make our point clear.

Let us now raise the question as to the meaning of the variable elements in the original set of postulates. It is clear that they do not refer to a unique set of things so that the meaning of the elements can not be specific in the sense of indicating uniquely the things referred to. But it is certain that the variable elements are not devoid of meaning, for they must always be such as to verify the quite specific general theorem with which we concluded the development of the doctrinal function. The meaning of the variable elements is ambiguous in the sense that more than one interpretation is possible; but it is a controlled ambiguity. It is the presence of this controlled, or limited, ambiguity in a doctrinal function which gives to it its great value as an active generator of doctrines. There must be an ambiguity of reference if a wide variety of interpretations is to be possible; but that ambiguity must be controlled if we are to be able to reach logical deductions. If the terms employed meant anything what-

ever, we could not use them to conclude to definite theorems—a thing which we have seen to be possible.

It is the set of postulates which controls the ambiguity. Whatever interpretations are possible, each of them must be such as to transform the postulates into true propositions. The postulates by their form restrict the truth-range of the resulting doctrinal function. They may permit it to have an infinite range, but all possible ranges—however they differ concretely—must have something abstract in common, namely, those properties in virtue of which the postulates become true for every interpretation. The postulates allow an ambiguity of interpretation but they control that ambiguity and hold it within certain limits.

## VI

In the reasoning through which the postulates are expanded into a doctrinal function the thinker is dealing with the variable elements in all their ambiguity. That he is not dealing simultaneously with all the interpretations is evident; for interpretations may exist, as we have seen, of which he is ignorant at the time when he carries out his arguments. Nevertheless he is clearly dealing with the whole doctrinal function at once, since every conclusion that he reaches will yield a true proposition when transformed in accordance with any particular interpretation. Perhaps we would not go far wrong if we asserted that the argument is made by means of the forms of the postulates and of the other propositional functions which are derived from them during the progress of the reasoning.

What we have now seen of the logical processes involved in the development of a doctrinal function from its postulates will enable us to subject a certain interesting theory of reasoning to an intimate criticism. In his *Psychology of Reasoning* (translated into English by W. A. Holl), Rignano sets forth this theory in the following words:

"Experimental verification and reasoning appear then to be *really one and the same process*, in the sense that the second is only the experimental proof itself, imagined instead of actually performed" (p. 78).

"It (reasoning) would seem to be nothing else, we repeat again, than a *series of operations or experiments simply thought of*, that is to say, operations or experiments that we imagine performed on one or several objects in which we are particularly interested, and that we do not perform actually because, by a series of similar experiments which have been really accomplished in the past, we already know their respective results" (pp. 81-82).

"Thus mathematics can be defined by saying that it is the science in which the merely imagined experiments, which constitute reasoning, are of a very general quantitative nature, capable of making the most varied physical phenomena equivalent in regard to the quantitative relations thus discovered" (p. 194).

"All deduction, even in the purest and most formal logic, is nothing else, we repeat, than a combination of merely imagined operations or experiments" (p. 199).

"The fundamental nature of reasoning, as a series of merely imagined operations or experiments, remains without change even in cases where an abstractness pushed to its extremest limit and an excessively complex symbolic form might at first sight succeed in concealing or even disguising this fact" (p. 207).

Here, in the words of Rignano himself, we have a compact statement of his theory of reasoning. In his opinion

the steps of a process of reasoning always consist entirely of operations or experiments which we imagine performed on one or more objects while we do not actually perform them because we have done similar ones before. It is not our present intention to assert that reasoning is never of this sort; but we shall maintain that it is not always the kind of thing here described, and in fact that the reasoning employed in developing a doctrinal function does not consist merely of imagined operations or experiments upon a set of objects.

Let us recall that the postulates contain statements about variables. Some of the terms are not specifically defined. The set of postulates has a great number of interpretations but the reasoning is valid once for all for every interpretation, notwithstanding the fact that it may be carried out (and actually was carried out in our case) while the reasoner himself is unaware of the existence of certain of the interpretations. It follows from this that the imagined operations or experiments of Rignano's theory could not have been carried out simultaneously on each one of the definite interpretations of the system of postulates. To the reasoner himself, even if he had all these interpretations in mind, it is clear that he is not thinking of imagined experiments on the objects in each of them, or even in one of them. Others also must surely be convinced when the reasoner, after developing the doctrinal function, finds an interpretation of it previously unknown to himself.

Therefore, if the reasoning employed in developing a doctrinal function consists of a series of imagined operations or experiments on objects, those objects can not be the totality of objects afforded by all the interpretations of the doctrinal function. What, then, can they be? Some of the terms in the doctrinal function are essentially ambiguous. They are capable of many interpretations. It

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does not appear to be possible, even in a simple case such as we have treated, to exhaust the concrete interpretations and to know that one has done so. It is necessary for the argument to proceed by the use of terms which are ambiguous and whose concrete range of ambiguity is unknown. What is known in the way of determining the ambiguity is the control afforded by the postulates themselves. It is this which prevents the doctrinal function from being futile and makes it in fact one of the greatest means of economizing intellectual labor.

It is a fluid material (with a controlled fluidity) that is employed in the reasoning connected with a doctrinal function. And yet the conclusions have the fullest validity and certainty. The logical processes employed in their development appear to be as secure as any known to the mind of man. They are clearly not imagined experiments on objects the results of which are known by previously made actual experiments. The objects are not defined in a way to make this possible. They are variables and only a part of the constants in the truth-ranges of those variables need be known while the argument is being carried out. (In fact, such arguments can be carried out in part—and accurately as far as they go—by one who knows no interpretation of the set of postulates which he is treating; later I hope to show this by setting forth the results of the requisite experiments, experiments that have already been performed in part.)

Whatever may be the range of validity of Rignano's theory of reasoning, it is clear that the development of a doctrinal function furnishes an example of a case where that theory is inadequate. The meaning of the processes can not be made clear on the basis of Rignano's theory. Then how shall we understand their meaning?

## VII

We shall make progress toward the answer to the last question by giving close attention to the implications of the fact that the postulates and theorems of a doctrinal function are propositional functions.

The reasoning is not made by means of concrete or sensuous images. To see that this is so, one needs only to examine the consequences of supposing that the concrete or sensuous images are carriers of the argument. Any concrete interpretation of the system of postulates necessarily has properties which are not implied by the postulates. Let us call those the extraneous properties of the given interpretation, these being extraneous in the sense that they are not demanded by the postulates themselves. We may call two interpretations essentially distinct when they differ in some of the extraneous properties belonging to them. That such essentially distinct interpretations exist for some systems of postulates is made manifest by the four interpretations already given for the system of postulates of this paper. They usually exist for systems of postulates. The argument used in the development of a doctrinal function can be made when the reasoner is still ignorant of interpretations which are essentially distinct from all of those which he knows—as is clear from the foregoing part of the paper. And yet the doctrinal function will become a valid doctrine when given any new interpretation.

Let us see how this could come about on the supposition that the reasoning is made by means of concrete or sensuous images. An extraneous property of a particular in-

interpretation will have a tendency to intrude itself into the argument. If it does so effectively some conclusion reached will depend for its validity on the extraneous property. It will therefore not be valid for an interpretation not possessing this extraneous property. But this is in contradiction with what we have already seen to be a characteristic of a doctrinal function, namely, that every propositional function in it will become a true proposition when interpreted in terms of constants belonging to the truth-range of the system of postulates. Hence no extraneous property of a particular interpretation can intrude itself effectively into the argument. How can this intrusion be avoided if one reasons by means of concrete or sensuous images? It can not be done if a single interpretation is carrying the burden of the argument, for it would then be inevitable that some conclusions would be reached which are special in the sense that they apply to the interpretation in consideration and not to the doctrinal function as such. This intrusion of extraneous properties can then be prevented only by playing one interpretation against another. If it is clear to the reasoner that he is not weighing one of the interpretations against another to prevent conclusions special to one of them. If he were safeguarding himself in this way it would be necessary for him to have in mind during the argument all the possible essentially distinct interpretations—and this we did not do in developing the doctrinal function of this paper. We must conclude that the reasoning is not carried by the concrete or sensuous images. In some way or other the postulates are effective to prevent the errors which would be inevitable if the argument were sustained by concrete or sensuous images.

It is necessary to guard against a possible error in the interpretation of the preceding paragraph. It applies to the arguments by which the theorems are established, and



to all these arguments. But it need not necessarily apply to all processes of discovering or sensing the theorems not previously known. It certainly applies to some of these processes of discovery, as is shown by experience in dealing with doctrinal functions. But one may sometimes be led to expect a theorem in a doctrinal function by observing its presence in one or more particular interpretations. The interpretation is an aid in discovery. It helps towards finding a conclusion which is held tentatively pending the exact argument by which it is established as a part of the doctrinal function; but the postulates themselves must carry the burden of the argument.

We may profitably pursue the digression a little farther. Suppose that one is trying to develop rather fully a doctrinal function of considerable extent and that he already knows several interpretations of the basic system of postulates. Now if he observes the presence of an important proposition in the field belonging to one of the interpretations he will naturally look to the doctrinal function itself to see if there is a precisely corresponding propositional function in it. If so he will be able to establish that propositional function by an independent and abstract argument. If there is no precisely corresponding propositional function he may nevertheless be able to find one closely akin to the proposition from which he started. In this way the interpretation helps in the development of the doctrinal function. Another interpretation may help in another; in practice, it often does so. The various interpretations afford various heuristic guides and in this way greatly aid the progress of discovery. Not only so; but one interpretation, through the intermediary of the doctrinal function, affords an aid to learning the propositions in another interpretation, even though these two interpretations should have little that is concrete in common. The inter-

action of the doctrinal function with its interpretations affords a valuable aid to discovery.

• Let us return to the question as to the meaning of the processes employed in the development of a doctrinal function. We have just seen that the reasoning is not carried by the concrete or sensuous images which may be attached to one or more interpretations of the system of postulates. In the previous section we saw that the argument does not consist of imagined experiments such as are contemplated in Rignano's theory.

It can not be that the validity of the argument is sensed by a happy intuition of properties of the variables in the postulates when the results of such intuitions are not formed into definitely established propositions or propositional functions. In dealing with concrete things which are very familiar one may arrive at true judgements concerning them by means which are not consciously logical in character and have a confidence in his results which approximates to that afforded by a strict logical analysis. But experience shows that that can seldom or never be the case in dealing with a propositional function. The variety of possible interpretations interferes with any such free course of intuitional procedure. By no such method could one ever be sure of covering all possible interpretations, to say nothing of the abstract situation involved in the doctrinal function itself. It can not be emphasized too strongly that the handling of the particular interpretations—even though it were possible to contemplate all of them and to know that one had all of them in mind—is a different thing from handling the doctrinal function as a whole. It may not be possible for one to get the whole force of this distinction without considerable experience in developing doctrinal functions and perhaps even with creating or constructing them, postulates and all. The doctrinal function is an entity different from any or all of its inter-

pretations; and the reasoning in its development is in some respects different from that which would be apparent in the development of any one of the doctrines to which the doctrinal function gives rise.

Does the reasoning in the development of a doctrinal function rest entirely on the properties specified for the undefined elements and relations? In a sense, yes. But it is important to notice clearly in what way this affirmative answer is to be qualified. One must not lose sight of the fluidity of the elements involved in the doctrinal function. A fundamental ambiguity runs through the whole system—a thing which gives the doctrinal function one of its chief values. But it is a controlled, and not an arbitrary, ambiguity. As we have seen already, it is the control which renders possible the reasoning in the presence of the ambiguity. The reasoning, in resting on the properties of the elements and relations, is modified by the presence of this essential ambiguity, usually infinite in the sense that an unlimited number of interpretations is possible.

It is probably most satisfactory to say that the reasoning rests primarily on the forms of the propositional functions employed. At the outset of the reasoning these are just the propositional functions contained in the set of postulates themselves. As the argument proceeds additional propositional functions are established and these may then be freely employed. In an extensive doctrinal function the explicit base of most of the argument is in these derived propositional functions. Of course they rest ultimately upon the postulates themselves. But the reasoning at each step rests necessarily upon propositional functions. As far as I am able to see from my own experience with such argument the reasoning seems to rest primarily upon the forms of the propositional functions; and this affords its main characteristic.

As I come near the conclusion of the paper I am more than ever conscious of the difficulty of making clear those aspects of the problem of meaning which are involved in the development of a doctrinal function. Perhaps it is impossible for one to sense the nature of this part of the problem simply by hearing or reading about what is involved in it; it may be necessary for him first to have considerable experience with the reasoning in the development of a doctrinal function—not as a bystander who witnesses the reasoning but as the agent who performs it. For many years I have been impressed with the remarkable nature of the arguments involved in such work. The impression grew from actual experience with them, and such experience may be necessary to an appreciation of the nature of the problem. As I think more about the whole question the more am I impressed with the necessity of this direct and active experience on the part of anyone who wishes to understand the nature of a doctrinal function and the meaning of the processes involved in its development. One does not come to an understanding of the processes by hearsay; he must insinuate himself into a possession of them by actually taking part in employing them with at most only a partial guidance. In some earlier articles in the *Monist* I have given other examples of systems of postulates. These are simple enough to serve for the purpose of inducting one into an understanding of the nature of systems of postulates and the consequent doctrinal functions.

But one will never understand doctrinal functions fully until he has constructed some for himself. To learn to do so may not be easy; but it is a necessary step toward an understanding of one aspect of the fundamental problem of meaning. No one can consider himself a master of the notion of doctrinal function until he knows how to construct simple examples having some elements of novelty

and real interest. Whoever considers this "an hard saying" lacks either interest in the problem or the resolution necessary to its clear understanding.

Some features of our problem emerge into a clearer light in the process of constructing a doctrinal function for one's self. In the nature of things it is impossible to bring this out explicitly in the article itself. If I should exhibit the construction of a doctrinal function I would merely put before the reader one more constructed for him. I could not guide him through the process of constructing one for himself. That he must obviously do on his own initiative. A useful system of postulates to think over as a guide to constructing one for himself is that which I have reproduced in my article on "The Structure of Exact Thought" (*The Monist*, January, 1924). During the composition of this paper I have constructed some new doctrinal functions, of simple character, so as to have the experience freshly in mind while writing; but it does not seem worth while to reproduce them. The system actually given is well known; it was chosen on account of its simplicity and its close contact with an important division of mathematics. But it is easy to develop doctrinal functions which are essentially isolated—at least if one understands them at all. It is the latter sort that the non-mathematical reader would find most convenient and most profitable for his exercise. The work is commended to anyone who desires to see clearly the problem of meaning under the color afforded by a system of mathematical postulates and the derivation of their consequences. A certain initiative of original or creative thinking is necessary, but the returns are well worth the effort.

R. D. CARMICHAEL.

## THE PRAGMATIC ABSOLUTE

IN the concerns of our everyday life we are all Pragmatists. In a large part of our conduct we simply are opportunists, and this is as true, or almost as true, of professors of philosophy as it is of manual laborers. Questions of the household budget are met in the "what is best on the whole" manner rather than in the absolutistic spirit. On January first we did not know certainly whether it would be better to spend more in 1924 for recreation and less for clothing. We had only experience to guide us to a decision. We do not know yet how our decision will turn out, and when the year is over we will not be able positively to decide, except in an approximate manner, whether we acted for the best. There are many factors in the situation, each and every one of which is an indefinite. If at the end of the year we are able to understand clearly what the results are, yet we will not be able quantitatively to assess the causal energy or inertness of the several factors.

Let us take another decision situation, perhaps on a higher level of seriousness. When one is called on in his adolescent period of development now finally to decide on his career, there is not infrequently a plentiful lack of certitude as to just what vocation one should honor with such selection. Yes, and when, mayhap, we in the plenitude of our powers are forced to decide, yes or no, what we are to do with that call to another position, have we any more resources of certitude than has the anxious sophomore with his problem of a vocation? In vain we look for a sure principle upon which we may base an inerrant decision. We

find ourselves reduced to a practical balancing of reasons for and against the proposed change, and this reason-matching game is likely to proceed until we find ourselves exhausted, rather than the supply of possible reasons which reason so easily, so tantalizingly, so tiresomely seems able to supply. No absolute guide is available here though the decision involved is serious. The final stage of such a decision situation oftentimes is that one throws reason and all its works to the winds and does the thing he feels for, much the same as his grandmother would have done. If it is true that no absolute is available in making the practical decisions of life, is it not then true that practically there is no absolute for us? But it is just the practice of life that counts, and what does not count here may rightly be counted out.

As I understand pragmatic philosophies, surely more than thirteen by this time, this is the gist of the reasons for rejecting an absolute, and proposing pluralism in some form. If, on examination, the above considerations prove conclusive, I, for one, am ready to embrace pragmatism, humanism, Dewian voluntarism, Nietzschean super-voluntarism, creative evolutionism, behaviorism, go-as-far-as-you-can individualism, as the final and ultimate philosophy possible to man, although those last two adjectives do have a rather absolutistic twang.

If one could prove the actual existence of an absolute that would, of course, settle the matter and leave pluralism nowhere, except included within the absolute. But I do not propose to embark upon this venture. Not a few have essayed this grand finishing off of philosophy, but it is not the philosophical fashion of our day. I reverence the great philosophies but their fates appall me. And perhaps, with the young Socrates of Plato's dialogue "Parmenides," I am afraid, afraid of falling into a bottomless sea of nonsense. In any case, my purpose here is a far humbler one,



namely, to examine whether an absolute be not necessary.

If an absolute proves useful to us in the conduct of our lives, if we find that we can do things by having an absolute, which otherwise we can not do, then as sensible empiricists we will surely adopt an absolute. If, in attempting to formulate a philosophy, we find that a concept of a final norm, or essence, or authority, or force, best serves to organize our thought and direct us in progress, then we as pragmatic philosophers will surely and cheerfully postulate the needed finality, nay, we will demand an absolute. And what is this but realism, this accepting what is forced on us?

Let us turn to the traditional departments of philosophical endeavor. Who can deny that in epistemology it would be convenient to possess a fixed norm of truth? Whether this norm were idealistic or materialistic in its character it would clear enfeebling doubt from our minds, and be most valuable as a time saving dispute-settler. As a comment I might add that I do not see how such a norm could be a neuter, for a neuter not only does nothing but it is nothing.

The pragmatic standard of truth is defective because it fails us just when we most need help. To say that a proposition is true if it works well in experience, or to the degree that it works well when tested, is not to get much further than the common sense level of rule of thumb empiricism. As reflective thinkers we are forced to go further than this and ask the meaning of "works well." We must ask for definition; "works well" in relation to what, for how long, and how often? It is not an answer to this difficulty to invite us to go ahead and test the questioned proposition until we are tired. Weariness and lapse of interest are not certitude, and they settle nothing except the fact that we are ready to give up the quest.

The fact that I find that a given proposition works well in an experience of mine, and in many such cases of experience, and that I am able to verify that other men, many men, agree with me, would seem to lead to the inference that there is something fixed and dependable either in my knower or in the thing known, or in both. The above comments apply no less to the demand of the realist that the knowledge act shall be a vital grasping by the knower of a real object. How shall it be known when we have gotten our reality, and how fully, unless we have a standard of success which shall arbitrate between our individualistic knowledge act and the independent object which we attempt to know? It certainly would be convenient here to have a standard of truth which one could fully trust. Even if we had to acknowledge that we did not possess unto perfection this absolute standard of truth, we could have more confidence in the whole knowledge process if we believed that such a standard existed. Such a faith would encourage us to strive to go on unto perfection. And is not every serious attempt to find truth a postulation that there is truth to be found and that we may attain it, however partial our success may be?

We require a standard of truth, one that will work when we most need it. Why not have it? And if any should denounce our procedure as an impious stretching forth of hands to snatch the fruit of the tree of knowledge and challenge our right to possess it, let us answer with Rooseveltian frankness, "I took it."

Surely no pragmatist can consistently impeach such a high-handed assumption of the absolute to our epistemological needs. We need but to answer, "In my necessity I did it." And if we content ourselves to use Kantian terms, and say we postulate such an absolute, we will mean, as Kant did, that we do this in no light-hearted spirit of

preference, but that by our very attempt to attain truth we, by this act, postulate an absolute truth.

Let us now raise the ancient metaphysical problem, and ask what is real. We need not demand to know just what is the ultimate nature of the real, but only whether there is something which is truly real, a something there, which all knowers will find when they know. Is the world which we attempt to know in any sense systematic, or is it just a sum of isolated and unrelated bits of stuff? No metaphysics has ever radically assumed this latter proposition, however pluralistic it may have proposed to be. Such an atomism would destroy not only the possibility of knowledge but also the possibility of objects, for there seems to be nothing simple in our world, mental or physical.

But once admit that reality is systematic at all and we at once have on our hands the same old problem of "how much." Perhaps it is just as systematic as we find it to be. Such an answer is hardly satisfactory, for it seems to imply utter subjectivism and to assume, without proof, that nature is no more systematic than are our attempts to know her. But all of man's advances in the knowledge of nature, and his subsequent dominion over her, have been in just the other direction, that of assuming that there is order in nature and that it is, therefore, necessary for us to find and conform ourselves to this objective order of reality. The realists are right at least in their contention on this point.

But once granted any degree of law, order, system in nature, where are we to stop? Of course, we do not stop at all. We go on investigating, experimenting, quietly assuming that the orderliness in things is sufficient and that it will hold out. No wonder the creative scientist is impatient when we ask him how he knows that the nature he is investigating is orderly. "Know! Why I assume that it is, and get results." And all of the workers in every

field of investigation do the same. Success is the result. Why quibble? Why not assume that this rich and faithful order will never fail us? And this tremendous assumption is just what we have implied in every step of our investigations, if we are productive workers and not merely timorous metaphysicians. Needing this faith in the dependability of the world order, on pragmatic grounds we have the right to assume the reality of such a world order.

We have said nothing as to what is the final nature of reality, except that, for us, it is orderly. In its final essence the Universe may be idealistic or materialistic. It might, of course, be both but I choose to discard the dualistic hypothesis, partly from lack of time to discuss it, but mostly because I do not consider it necessary to champ over that old straw. I believe that absolute idealism and materialism are the real opponents in the field of philosophy today. Pragmatism, the various realisms, the more or less idealisms, seem to me either to shirk the metaphysical problem or else refuse to go through with it. If one believes that the world, including man and all his works, are mechanistically arranged, then one is a materialist, but if one believes that the world order is intelligently purposive, one is an idealist. One may not be able to prove, perhaps it can not be proven, which of these antithetic propositions is true, but one is justified in choosing between them. If the mechanistic hypothesis best fits the facts as our reason knows and understands them, and most satisfactorily arouses our emotional reaction and with this our will, then materialism it is for us. If on the other hand the teleological hypothesis makes the world more understandable to our reason, if it better arouses our emotions and causes these to drain into actions of profit and honor, and into appreciations of beauty, then idealism is the philosophy for us. Which philosophy works best? That one we choose.

If now we turn to religion we find the assumption of the Pragmatic Absolute as an actual working method. Here one does not have to ask if it would be convenient to have an Absolute as a background assumption, for such an assumption has already been made and is in fact the chief organizing concept of this very extensive portion of the life of mankind. The practical postulation of an Absolute is courageously done, in the Kantian manner of a postulate necessary to the rational life of man, and not as a timid hope. To the man of dynamic faith the rationality of the world in which he lives and the dignity and worth-while-ness of the life which he lives depend on the reality of a God who is limited neither in his power nor in his love. Such a believer need not attempt to prove the existence of the God he needs; he asserts his existence. So far from his faith being a system of proven beliefs the active religious man never questions his faith except in moments of weakness and defeat. His faith is what he lives by and not the pindling product of logical exercises. He asserts his God and stands by the consequences. All values stand or fall together. Either God is, and heaven therefore possible, or else all is hateful, meaningless night. The religious man in his dire need passionately asserts God and wills heaven.

I am not unaware that much theology has been written to prove the existence of a God, infinite in all his attributes. It is significant, however, that the religious mystics have had little use for theology. They have had other reasons for their faith. Even the theologians in their practical religious life have trusted a living God rather than a logical deduction. See Calvin, in spite of his "Institutes."

In religion there is an antagonism between theology and experience, which is a part of that wider antagonism between reflective thought and practical action. It is not unusual to hear advocates of the present-day program of

sociologizing religion speak slightly of "mere dogma." It is action which they wish, reforms of the social structure in the direction of greater social justice, as they view it. Such reformers seem impatient with creed making, not so much because they disbelieve the formulations of such creeds, as because they fear that emphasis on beliefs will draw away attention from more vital and pressing problems of social readjustment.

There is a somewhat similar type of religious leader who fears that credal emphasis will be substituted for personal righteousness. The emphasis here is individualistic and not social, but in both cases it is the moral aspect of religion which is stressed. Both are voluntaristic and not rationalistic. These are practical interpretations of religion and in them is felt the need of but few and simple credal assertions. Such beliefs as they feel to be necessary are assumed, and little time is lost in attempted proof. It must be that God is and that He is a reward of them that diligently seek Him. God is wise enough to know what is best, powerful enough to bring it to pass, and He is "Our Father."

The practical man of religion asserts that there is a universal power that makes for righteousness, and he boldly takes sides with this beneficent force. All faith is a venture, a dare flung to destiny. The religious soul stakes all on its highest expectations. It is all or none. It is just from its glorious daring that faith gets its power over men. One desires all things of high character and of good report to be true, for these alone make life worth the living. Why not risk all on what is most precious to us, and live accordingly? If one's personal efforts fail there is yet the Absolute, however conceived, as an all-enfolding protection. The great cause will win though I see not the glory of the Lord in the land of the living.

As I interpret religions, at least developed religions, the Absolute is sure to be there. Even in the Persian dualism Ahura Mazda, the god of light, is finally to have the victory over Angra Mainyu, the god of darkness. The dualism in Christianity is only superficial, for the devil's power is limited and will suffer a dramatic end. William James' pluralistic world, in which God is but one of the forces, is not convincing, from the religious point of view. The "tough-minded" attitude, which he recommends, is not the attitude of faith. We are invited to "pitch in" and help out the spiritual cause which we approve, for it may win by our efforts. This sounds too much like an invitation to join a Spanish-American revolution, in the spirit of Don Quixote. It is not the risk which one most fears but the futility of the thing. If we win, what have we won, and for how long? We need no genius to point out to us that in the life of our every-day experience there is plurality enough. But such plurality is of the very essence of the evil which we must overcome, and it is not very inspiring to be invited to worship one element in this confusion.

I can not bring myself to say hard things about H. G. Wells' finite deity. This limited deity is a product of the influence of the Great War on a sensitive mind struggling in the midst of a maelstrom of horror. Mr. Britling, another name for Wells and for tens of thousands of other British fathers, could not tolerate the idea of an all-powerful and benevolent overruling Providence as he sees his ideals swept, one after another, into abysses of horror which seemed to have no nethermost depth. He falls back upon the idea that his god hates all this even more than he can and suffers from it with an intensity that man can not conceive, yet this deity can not stop it. He saves God's goodness at the expense of His power. But the war has passed and its psychology will pass, and man will find that



his God must have power adequate to sustain the values necessary to worthy life.

Surely the boys in the trenches were nearer to truth in their falling back upon a primitive faith in fatalism. They saved their sanity and morale in that unaccustomed savage life by narrowing their creed to a comforting belief that they would not die until their time came. No shell could kill them unless it had their number on it, and nothing could save them if it had. So why worry? They were in the hands of destiny, and nothing that they could do would change that unaltering order. Fatalism is the philosophy of the close corner. Overwhelmed by titanic brute forces, men in their dire extremity place the responsibility for the outcome where it belongs, on an overruling order. In his utter helplessness man escapes the ruinous shame of individual isolation by identifying himself with a majestic, all-inclusive order. He does not demand that this order of things shall give him personal attention, but only that it shall include him. Such a fatalistic faith is not a faith of the highest type, but it is more normal than any highly sophisticated rationalization which attempts to save God's face in time of disaster by denying Him the power to help Himself.

The above analysis is an attempt to define the essential element in the religious attitude toward the Universe. This religious interpretation of total reality includes the assertion that it is a whole, in some sense, and such a whole that man can safely trust his dearest interests to it. The practical religious man, or any religious person when in the practical mood of action, is not interested in proving the existence or in minutely defining the attributes of his object of worship. He assumes the existence of a God worthy to be trusted and proceeds to avail himself in his necessity of the aid of such an all-powerful helper. Such people are busy with life, and to be asked to stop in mid-career of their

struggle and to perform a dialectical exercise of proof strikes them as of all things most futile and out of place. The mood of battle does not harmonize with the critical atmosphere of reflective thinking. So it has come about that the concept of deity of the theologian and the philosopher, and the God of men in the urgent practice of living scarcely seem the same being.

Unless, however, we are forced to it we will not willingly acknowledge that this apparent dualism between the voluntaristic necessities of man and his reflective interests is really an irreconcilable dualism. Is not man's reflective activity, however abstract, a real activity after all? Why then should the reason reflective and the reason practical differ fundamentally in their essential interests, methods or conclusions? And when one examines the great products of reflective thinking one may justly doubt that they do so differ. Calvin really assumes his deterministic Deity and proceeds to map his attributes, and to describe what such a God would necessarily do. He needs such a God to make the world rational, to give man dignity and to furnish the church with a task. All follows, very logically indeed, from the initial assumptions as to the nature of God. This procedure is not alone rationalistic, in the reflective sense, for it does not lack a voluntaristic element, at least in its initial stage.

Royce, in his major work, *The World and the Individual*, reverses the procedure described above. He attempts to prove the existence of the Absolute. And yet is not what he really proves that the Absolute is necessary to complete thinking about man and the world in which he lives? We are told that to deny truth is to assert the existence of a truth, namely the truth that truth does not exist. To deny is to affirm, which is as much as to say that reason can not get on without a sure standard of reference. The first act of reasoning is in itself an assumption that there is a something to know and that this can be known. The will to

know is the assertion of the possibility of truth. But truth to be truth can not remain in the fragmentary, empirical stage. Reason itself demands that we pass on to a complete whole of understanding. But how do we know that such a whole exists? Well, reality proves to be a re-representative system. No bit of reality, as we experience it, exists in self-sufficient isolation. It is only in relation that the fragment gets meaning, and any relation carried out points inevitably to a whole of well-ordered relationships. The monad mirrors the Universe.

In the labored Supplementary Essay, at the end of Volume One of *The World and the Individual*, Royce meets the acute criticisms of Bradley, not by proving the existence of the Absolute, but by showing that the concept of the Absolute is not self-contradictory and that it does not exclude the reality of the individual. In other words, reality may be a whole and yet the parts may be eternally significant. But what is really attempted to be proven here is not that the Absolute exists but that the Absolute is possible. Royce's argument does not attempt a formal proof of the existence of the Absolute, but rather that the existence of the Absolute is necessary.

When we attempt to know a bit of reality we find that we must understand the relations of this one item, and just in proportion as we are successful in our knowledge act we are lead on toward a whole of truth. In trying to fulfill any purpose of ours we find that just to the degree in which we understand the true meaning of this purpose and get it fulfilled that in that measure we are led on to an all-inclusive purpose. Here we arrive at an Absolute, as the last step in our process, rather than assume such an Absolute as our initial act. But, first or last, the Absolute is had by an act of assumption, and this is no less true of Royce than it is of Calvin, though their methods are in different degrees convincing. For Royce, and for a great

company of like-minded thinkers, the world must be intelligible, truth and right must be secure. Nothing else is fit to be believed. Much experience, perhaps all successful experience, leads on toward a whole of experience, where all is explained and fulfilled. Let us take the hint. Our reasons demand this unity; our hearts desire with a great longing this everlasting safety; the life of reflective thought and the life of practice alike prosper best when it is assumed that reality is trustworthy; why not trust it to the uttermost? Yes, we affirm the Absolute.

This decision is not a formally logical conclusion. It is voluntaristic rather than reflective, and it does not lack an emotional tone. The only logic here is the logic of all the circumstances, that of boldly daring to assert the best conclusion which we are capable of formulating. In the last analysis all complete philosophies do just that: they affirm that final interpretation which best suits the philosopher concerned. I do not use the term "suits" lightly, as a mere personal preference, but in its gravest sense of fitting best what is known by the thinker and the practical doer of deeds alike.

And why should we not postulate an Absolute, philosophers and practical religious men alike, if an Absolute best solves our problems of thinking and of living? Ours is a pragmatic Absolute.

In the world of ethical action as well as in the philosophies of right it is not difficult to point out that a dependable standard is wanted. I have already touched on but not specifically discussed our requirement of such a standard of reference. Here, as in other fields of vital interest, the finality of the authority is really postulated. Proofs in plenty have been proposed, of the existence of such an authority, but the decisive factor has been that man has found that he had to have, both in his personal life and in his society, an unquestioned authority; and he has taken it.

But rather than enter upon a discussion of the general field of ethics I wish to take a single phase of our social life, namely a present-day political episode, and see how the pragmatic Absolute applies here.

Our present national governmental situation illustrates well the pragmatic necessity of acknowledging inviolable and absolute principles.

It will not work well to trust even the selected "best minds" each to adopt what he thinks the necessary measures to further what he considers the best interests of the country. This is indeed to have a government of men, i. e., of pluralistic purposes, rather than a government of laws, i. e., of unified social purpose. In the practical affairs of government, especially in so large a body as our federal government, it is necessary to acknowledge a source of the last word. This final arbiter in executive affairs presumably might be the consensus of opinion of the cabinet, or it might be the decision of the president. This is necessary for efficiency in ordinary affairs of government, but however this final arbiter may be constituted, he or it must acknowledge a yet superior authority, i. e., the authority of the principle of right. This principle of right is supposed to be embodied in the constitutional and other law of the land. But all laws, as well as all administrative acts, must be justifiable on the principle of the good of all the citizens of the nation. Congress can, evidently, investigate as to whether this general principle is being lived up to by the executive branch of the government, and the Supreme Court can, when rightly approached, say an effective word on this same point, as well as upon the legislative acts of Congress.

The present controversies raging within the executive and the legislative branches of our government, and between these two, constitute an inconvenient illustration of pluralism of purposes, the result being that the business of

government is not properly going forward. The root of this governmental confusion seems to be that officers of the government have used their offices for their own personal interests, for the benefit of their friends, personal and political, to favor special groups which are determined to possess the nation's domain or to evade and even defy its laws, and for partisan ends. The trouble is the lack of a controlling purpose strong enough to direct all of the branches of the government into the channel of public service. Such a unitary purpose alone can cause the government again to function with the necessary strength and justice, and make possible the reassurance of the confused citizens that their government is still their own.

We started out to examine whether an Absolute were not necessary, on strictly pragmatic grounds. It was not difficult to point out that great numbers of men, and these apparently the most effective, have assumed an Absolute as the background of their activities. If the Absolute has worked so well, why object to adopting it? We have observed that the creative activities of man, whether in science or in practice, demand a standard of reference. The workers, who have brought us along the way called progress, assume that they have this standard in reality and that it is trustworthy. They assume this complete wholeness of their world by their very acts, and this seems no less true of the thinkers than of the doers. On what grounds shall one then object to the Absolute? Certainly not on pragmatic grounds. We take, therefore, the liberty of recommending to those empirical spirits, who will be practical at all costs, the adoption of the Absolute, a brave Absolute which will work all around the circle of necessary interests. If the re-introduction of the Absolute in this manner is annoying, we are sorry, but after all ours is a Pragmatic Absolute.

EZRA B. CROOKS.

## INTUITION AND BEAUTY

**T**HE term intuition has been used in a great variety of different senses, not only in popular speech, but also by philosophers, as for instance, to mention only three names among many, by Kant,<sup>1</sup> by Croce, and by Bergson. The task of classifying, or even of enumerating, these different usages is beyond the scope of this article, but an attempt will be made:

I. To discredit the view that a judgment predicating beauty is rightly to be classed among those unconscious deductions which are by some writers termed intuitions.

II. To define intuition in the only sense in which it seems to me desirable to use the term.

III. To define the referend of a judgment predicating beauty.

IV. To draw attention to some respects in which intuitive judgments predicating beauty (i) resemble, and (ii) differ from certain other intuitive judgments; and

V. To discuss the predicate of a judgment predicating beauty.

<sup>1</sup> More accurately, of course, by some of Kant's translators.

<sup>2</sup> The term unconscious will be used throughout this article to cover any psychic phenomenon of which its owner is unaware. In other words, it will be used to cover both those psychic phenomena which are unintrospectible and also those which are introspectible but which are not actually introspected. The distinction between these two classes, though very important in certain discussions, does not seem sufficiently relevant here to justify the clumsy terminology which its preservation would involve. Were not "unconscious" used in several passages which will be quoted, "unintrospected" would perhaps be preferable; but the use of one term in two senses—when the difference between the two is irrelevant—seems less undesirable than the use of two terms in the same sense.



## I

The process of unconscious inference, sometimes called intuition, has been noticed at least from the days of Aristotle. Dr. John Brown in a footnote to his essay on "Happy Guessing" in *Horae Subsecivae*, quotes "a Balliol friend's" comment on the term εὐστοχία, which Aristotle uses to cover what we might describe as "hitting the mark successfully, reaching to the end, rapid and as it were intuitive perception of the truth." "But," comments Brown's friend, "when Aristotle says that this faculty is not guided by reason, he does not mean to imply that it grows up altogether independent of reason. . . . On the contrary, no faculty is so absolutely the child of reason as this faculty of happy guessing. It only attains to perfection after the reason has been long and painfully trained in the sphere in which the guesses are to be made. What Aristotle does mean is, that when it has attained perfection, we are not conscious of the share which reason has in its operation—it is so rapid that by no analysis can we detect the presence of reason in its action."

"You may see," Doctor Brown tells us in the same essay, "the sense, and neatness, and pith of that excellent thinker . . . Miss Austen, when she says in *Emma* 'Depend upon it, a lucky guess is never mere luck, there is always some talent in it'—talent here denoting intelligence and will in action."

The following passage from a letter of John Stuart Mill's is worth quoting in this connection:

"I have long recognized as a fact that judgments really grounded on a long succession of small experiences mostly forgotten, or perhaps never brought into distinct consciousness, often grow into the likeness of intuitive percep-

tions. I believe this to be the explanation of the intuitive insight thought to be characteristic of women; and of that which is often found in inexperienced practical persons who have not attended much to theory, nor been often called upon to explain the ground of their judgments. And I should agree with you that a mind which is fitted by constitution and habits to receive truly and retain well the impressions made by its passing experiences will often be safer in relying on its intuitive judgments, representative of the aggregate of its past experience, than on the influences that can be drawn from such facts or reasoning as can be distinctly called to mind at the moment."

The above passage is typical of many in English literature<sup>3</sup> and writings more specifically psychological, from the days of Frederick Myers onwards, have abounded in examples of intuition, or of what Myers himself described as "subliminal mentation." The phenomena of post-hypnotic suggestion furnish illustrations perhaps as striking as any that could be desired, but they are probably too familiar to need more than mention here, and it will be well now to pass on to the special subject of our discussion, namely, judgments of beauty and their relation to unconscious deductions.

There are those who maintain, or at least appear to

<sup>3</sup> And, without going beyond our own experience, we can recall occasions on which in talking with our friends "thought has leapt to wed with thought ere thought could wed itself with speech." It is enough in the case of those with whom we are in close mental contact for us to hear, without knowing that we hear, an intonation, a tremor, the scarcely stirring stir of a sigh, and their sorrows are bare to us; we answer their questions before they have asked them; we anticipate the expression of the needs they half recognize; we smile at their unformulated witticisms; and we leap to the defensive before they have aimed their accusations. But here we are brought up against the baffling question of "contact between minds." I have written as though, in this swift intercourse, unconscious inference were necessarily involved. But is this indeed the case? Or is the pronoun "we," as Mr. Delisle Burns suggests, significant of a unique reality? Is "we" something other than a mere abbreviation for "I plus you"? Can we indeed enjoy the mental processes of other people as directly as we enjoy our own? It would perhaps be satisfying to establish a metaphysical basis for the prejudice in favor of the first person plural, but the question is hardly relevant to our enquiry and may be neglected. For whether this flashing insight into the minds of others is deductive or not, there can surely be no doubt that unconscious inference occurs in other connections, for instance, in the rapid mathematical calculations of certain specially gifted people, and our enquiry is simply whether or not our intuitions of beauty can be reduced to any kind of unconscious inference.

maintain, that all judgments predicating beauty can be reduced to unconscious deductions. There is Mr. Charles Marriott who asserts that "aesthetic appreciation depends upon the sense, which may not be conscious, of practical problems effectively solved." Mr. Marriott appears to maintain that when we deny beauty in the case of a stone cross that is neither Celtic nor of elongated shaft, we are drawing an inference from the unconsciously recognized premise: "the grain of stone cannot, as can the grain of wood, run vertically in the shaft and horizontally in the arms."<sup>4</sup>

It would appear that certain experimentalists in the psychology of beauty maintain a position somewhat similar to that of Mr. Marriott. It is at least possible so to interpret one of the pioneers in this branch of psychology, Mr. Grant Allen, who tells us that it is his object "to exhibit the purely physical origin of the sense of beauty and its relativity to our nervous organization."<sup>5</sup>

Again, we may consider the view of Professor Ward,<sup>6</sup> who quotes with approval Preyer's analogy between

<sup>4</sup> "Let us take a symbol with which everybody is familiar, . . . the cross. As carried out in wood, that is to say in the form of two crossed sticks or planks, apart from any religious meaning that it conveys, most people find the symbol aesthetically satisfactory if the proportions are reasonably good. But, carried out in stone, in the same form and proportions, the symbol becomes unsatisfactory. It seems to need modification. In practice the modification is generally made. The most popular forms of the cross in stone are either the Celtic, or 'wheel' cross, or else the tall form, common in village crosses, in which the shaft is greatly elongated and generally tapering from the base, and the arms are reduced to comparative insignificance.

The reason seems to me quite simple. In the wooden form, whether the two sticks or planks are mortised or merely nailed together, the grain of the wood, though it may not be perceptible to the eye, is felt to run horizontally across the arms. The wooden cross explains itself as a structure. Repeated in stone the same form does not explain itself. Since it is inconceivable that the grain of the stone can run vertically in the shaft and horizontally in the arms, the impression given is that the cross has been punched out of the stone in an arbitrary manner in disregard of its nature and substance. . . . In the Celtic cross an amount of stone is left, in the shape of the 'wheel', which not only strengthens the structure but, so to speak, contradicts the method of construction employed in wood . . . and in the tall cross the same results are obtained by evasion; by making much of the shaft and little of the arms."—*British Journal of Psychology*, Vol. XI, Part 1, October, 1920.

<sup>5</sup> *Physiological Aesthetics* (1877), p. 2.

<sup>6</sup> *Psychological Principles*, Chap. X, p. 2.

awareness of a discord and the confusion of "trying to reckon up a sum in one's head and failing because the numbers are too high." "Those colors," continues Professor Ward, "yield good combinations that are far apart in the color circle, while those near together are apt to be discordant. The explanation that one arrangement secures, and the other prevents, perfect retinal activity seems on the whole satisfactory." Professor Ward apparently suggests that the judgment: "This is beautiful," is based upon an unconscious inference from the unconsciously formed judgment: "My retinal activity is perfect."

The researches of experimentalists in physiological and psychological aesthetics are obviously important and valuable;<sup>7</sup> but if certain experimentalists are indeed claiming that judgments of beauty may be reduced to unconscious deductions from arithmetical and physiological premises, it is surely desirable that their claim should be questioned. The first point which is insisted upon in this article is that beauty is not a measurable quality of sense-data, and that a judgment predicating beauty cannot be deduced, however unconsciously, from such premises as "the vibrations of this note are in such and such a ratio to the vibrations in that note," or "the grain of the shaft of this cross runs at such and such an angle to the grain in its arms." Croce is right when he asserts that "Art is not an exercise of unconscious arithmetic."

It is possible that the passages quoted from the writings of Mr. Marriott, Mr. Grant Allen, and Professor Ward do not fairly represent their position. If their claim is merely that certain arithmetical or physiological phenomena are invariably present when beauty is predicated, it may be freely admitted that the claim is apparently valid

<sup>7</sup> Among the most interesting of such researches now being carried out are those of Professor Scripture (Professor of Experimental Phonetics in the University of Vienna) who by using a delicate instrument is enabled to count and analyze the vibrations in the poet's voice when reading his poems aloud.

and certainly interesting. But it seems unfortunate that Professor Ward and Mr. Grant Allen—if they do not mean what they have been interpreted as meaning—should have used the terms “explanation” and “reason” in the passages quoted.

## II

If a judgment predicating beauty is not an unconscious deduction, in what sense, if any, can it legitimately be termed an intuition?

I wish to define an intuition as an act of cognizing what is self-evident (noting that what is self-evident always might be, though comparatively seldom is, formulated in a self-evident proposition);<sup>a</sup> and I wish to define the self-evident as that which must be simply accepted, and cannot be inferred, or deduced. That a self-evident proposition appears to us to be true can never be an adequate ground for asserting that it is true; intuition can never be an alternative to reasoning. A proposition would obviously not be self-evident if we could say of it: “I cannot think otherwise and therefore it is true,” for we could then deduce its truth from another proposition, namely: “I am convinced that it is true.” That a proposition inevitably appears true to us may be the cause of our asserting it, or, in other words, it may be the reason why we think and say that it is true, but a “reason why,” in this sense is something very different from a rational ground. A rational ground provides the reason for a proposition itself being true. Nothing whatever can take the place of reasoning when we are establishing the truth of a proposition; intuition can only furnish psychological grounds for our *holding* certain propositions to be true.

<sup>a</sup> The point of this will be made apparent in Section IV.

Thus it is impossible to believe otherwise than that "two things which are equal to a third thing are equal to one another." The proposition is self-evident and must be accepted by any one who grasps its signification. But it is equally impossible to give any rational grounds for the truth of the proposition.

If, however, we assume the truth of certain self-evident and of certain empirical propositions, then we shall be justified in maintaining, not only that certain other propositions appear to us to be true, but that, given the truth of these self-evident and of these empirical propositions, those other propositions *must* be true. For instance, assuming the truth of the self-evident proposition: "Any two things which are equal to a third thing are equal to one another," and of the empirical propositions: "Tom is the same height as Mary," and "John is the same height as Mary," the truth of the proposition: "Tom is the same height as John," may be established on a rational basis. The truth of validly inferred propositions is, hypothetically, susceptible of proof; the truth of intuited propositions is not in any way susceptible of proof.

If this account of intuition were accepted, it would be well to abandon the term when referring to unconscious inference. The two acts are in sharp contrast and to call them by the same name can only lead to confusion of thought. The act of cognizing a self-evident proposition (or that which may be formulated as a self-evident proposition) is as different as can be from a process which involves reasoning "so rapid that by no analysis can we detect its presence."

The above account of intuition is in the main a paraphrase of *Principia Ethica*, Chapter V, page 143, seq. Since I am not certain that my interpretation would be accepted by Doctor Moore, I have deliberately stated in my own words what appears to me the correct view; and

I have refrained from attributing it to him, not, of course, because I wish to claim what is his and is valuable, but because I do not wish to attribute to him what may be worthless and my own.

### III

It is important to distinguish between an act of judging, a judgment, and the content of a judgment. In the following pages the term intuition of beauty will be applied to the act of cognizing beauty, whereas the judgment (which may be formulated in the proposition: "This is beautiful") will be termed not an intuition but an intuitive judgment of beauty. By the content of a judgment is meant the referend plus that which is predicated of the referend, and by the referend is meant that to which the judgment refers.

The referend of a judgment of beauty is a complex whole one of the constituents of which is always either a natural object or a work of art. In either case part of the apprehension of the person passing the judgment seems to be another constituent of the complex whole upon which judgment is passed. I here use the word apprehension loosely to cover the person's reaction to the natural object or of the work of art.

By a natural object is meant a mountain, a flower, a haystack, a cloud, a hut, a bird, a stream, an animal, a human being, or anything else that we can be directly aware of through our sense organs, and that has been either not modified at all by men, or modified from motives apparently utilitarian.

It appears to me probable that when a natural object is a constituent, the person's awareness, in so far as it is aesthetically relevant, (i) is primarily sensational, not perceptual, by which I mean that images are not necessar-



ily involved but only sense data, and (ii) is accompanied by a certain emotional experience which is a constituent of the complex whole on which judgment is passed. This emotional experience I call the aesthetic emotion; and the emotional experience plus the awareness of the object I call the aesthetic impression. It is not easy to be sure whether this aesthetic emotion is itself simple or compound. Is it a primary and unanalyzable emotion, which is accompanied by other emotions, or emotional states, or is it a compound emotion of which, for instance, awe is an element, or joy? Whichever be the case, the aesthetic emotion must, I think, be distinguished from another emotional experience which often accompanies the apprehension of a natural object, namely, that arising from the inability to express the aesthetic impression of the moment.

We may now briefly consider the case when a work of art is a constituent of the referend of a judgment of beauty. By the term work of art is meant the adequate expression, in a medium, of an impression which has been apprehended by the artist as worth while in itself. When a work of pure art is one of the constituents of a complex whole which is the referend of a judgment of beauty, the other constituent will, it appears to me, be the imaginative apprehension of the person passing the judgment, and by this I mean that certain of his images (namely, those which are artistically relevant) will be part of the complex whole on which he passes judgment. It is clear that to read a poem (or to look at a statue) may evoke images that are artistically irrelevant. For instance, a medical student, when reading Wordsworth's *Sonnet on Westminster Bridge*, may find himself baulking at the image of a palpitating red mass that spoils for him the line: "And all that mighty heart is lying still."<sup>\*</sup> Only certain images will be constit-

<sup>\*</sup> The illustration is taken from C. W. Valentine's article: "The Function of Images in the Appreciation of Poetry," (*British Journal of Psychology*, Vol. XIV, Oct., 1923).

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uents of the complex whole of which beauty may be predicated, and those images will be such as are involved in the adequate apprehension of the work of art. When a work of art is adequately apprehended, imagery and sense-data will unite in a single complex whole of which beauty may be predicated.

But a complex whole of which beauty may be predicated will not, when a work of pure art is one of its constituents, include any of the emotional experience of the spectator. The spectator will have an emotional experience while he is apprehending any work of art (and in the case of mixed art he may also have aesthetic apprehension of certain constituents) but such an experience will not in itself be a constituent of the referend of his artistic judgment. The extent to which a man has emotional experiences when apprehending a work of pure art probably bears no relation to his capacity for passing artistic judgments.<sup>10</sup> Of two art critics one may be highly emotional, the other cold; although the former may find it much harder than the latter to distinguish his would-be from his genuine artistic judgments, yet the capacity of the two critics for passing genuine artistic judgments may be precisely equal. The only emotional experience entering into a complex whole of which a work of pure art is another constituent, is the emotional experience of the artist; and his emotion is not a present but a "distanced" one.<sup>11</sup> To quote the familiar phrase of Wordsworth's, the artist's emotion has been "recollected in tranquillity."

This difference in respect to emotional experience appears to be involved in the fundamental distinction between nature and art. Nature—unless indeed we assume the existence of a personal creator whose creative activity

<sup>10</sup> We may perhaps hazard a guess that should any correlation be established, it will be inverse.

<sup>11</sup> *British Journal of Psychology*, Vol. V, Part 2, June, 1912. E. Brough. "'Psychical Distance' as a Factor in Art."

is exclusively artistic in character—is not the expression of an impression which has been apprehended as worth while in itself. When we pass an aesthetic judgment upon a complex whole of which a natural object is a constituent, we pass a judgment which depends upon our own personal act of apprehension and its accompanying emotion, and is in no way dependent upon any previous act of apprehension on the part of another person. But when we pass judgment upon a complex whole among the constituents of which is a work of art, our judgment is dependent upon a previous act of apprehension on the part of the artist, and upon his aesthetic emotion; in other words, it is dependent upon the artist's original aesthetic impression. Our own personal act of apprehension is secondary and incidental, and our accompanying emotions are irrelevant.

We may now consider a case of a complex whole which includes among its constituents both a natural object and the images of the person apprehending it. In this case the images involved are not perceptual in the sense that they are perceptual when they unite with sense-data to form a whole,<sup>12</sup> and we are enabled by their aid to say: "That is a cow," or, "That is a man," when the sense-data alone would give us merely a red or a black patch. These aesthetic images play no part in helping us to recognize objects as objects. They do not help us to say *what* "this" is, they help instead to give us a new "this." The point may perhaps be best made by means of illustrations. For instance, no one who is familiar with Whistler's paintings can fail in a London street, steeped in the peculiar luminous blue of late twilight, to apprehend a "this" different from what he would have apprehended had he never seen a Whistler picture. Again, it is possible, as I know from experience, to visit a picture-gallery and to exclaim when looking at a painting by John Nash: "The sky couldn't look like that!" only to be confronted, immediately on passing

into the street, by a sky that did "look like that." But for John Nash's picture, however, I should not have seen it so. Either I should have had no aesthetic impression at all—simply should not have noticed the sky—or I should have had a very different aesthetic impression from that which as a matter of fact I did have.

In these cases it seems clear that the constituents of the complex wholes upon which judgments of beauty are passed include both natural objects and images derived from previous apprehension of works of art.

There seem to be other cases in which the referend of an aesthetic judgment includes among its constituents both a natural object (or, of course, natural objects) and the images of a person apprehending it. I refer to the cases of a spectator apprehending aesthetically either (a) a work of mixed art, that is, a work of art among the constituents of which are natural objects (in the sense defined), for example, a scene from a ballet—say, *The Good Humored Ladies*, or (b) what I should like to call a reproductive-aesthetic construction, for example, a "nature poem" by a writer who is not an artist but who is able to construct verse capable of stimulating the reader's reproductive imagery and so of provoking an aesthetic impression.

We must now enlarge our description of the referend of a judgment of beauty, and sum up this section in the statement that every judgment of beauty refers to a complex whole among the constituents of which are either (i) a natural object and no images, (ii) a work of art and artistically relevant images, some of which may be perceptual, (iii) a natural object and images derived from a work of art, or (iv) the images of the spectator and (a) a natural object which is itself a constituent of a work of art,

<sup>12</sup> I am aware that this is a crude account of perception; but I have allowed it to stand because I merely want to insist that the aesthetic images to which I am referring are not the same as those factors, of whatever nature they may be, which enable us to attribute *meaning* to sense-data.

or (b) a construction capable of provoking an aesthetic impression.

Before passing on to section IV, I will suggest two further considerations. First, that when a person passing an aesthetic judgment is himself a painter or an art-critic the third of these classes is in his case likely almost entirely to supplant the first. And, next, that images derived from what I shall, in Section IV, describe as a work of pseudo-art, may intervene to prevent the adequate apprehension of a natural object. We may, for instance, find it difficult to pass a genuine aesthetic judgment upon a silver birch tree if we have been for long familiar with a reproduction of McWhirter's "silver queens."

#### IV

We may now pass on to consider some of the respects in which intuitive judgments of beauty (i) resemble and (ii) differ from certain other intuitive judgments.

It may be well to note in introduction that self-evident propositions appear to fall into three classes, namely:

1. Logical (inclusive of mathematical), which assert a relation of universals;
2. Ethical, which assert that a particular referend is, or is not, good;
3. Aesthetic and artistic, which assert that a particular referend is, or is not, beautiful.

There is, of course, an important difference between judgments of the first class and judgments of the second and third. It is sometimes stated that logic is concerned with truth in a way parallel to the concern of ethics with goodness and of aesthetics with beauty. But truth can be predicated of propositions only, whereas goodness and

beauty can never be predicated of propositions. The propositions: "This is beautiful," and "This is good" may be true propositions; but no proposition is either beautiful or good. A logical act of judgment which predicates truth can only occur when a proposition has been formulated and stands as referend of the judgment; but acts of judgment predicating goodness or beauty may occur without any proposition having been formulated. The majority of judgments which predicate beauty are, of course, never formulated.

We are not in this section at all concerned with ethical and only indirectly with logical intuitive judgments; our direct concern is with intuitive judgments that predicate beauty.

(i)

Both logical intuitive judgments and intuitive judgments predicating beauty are immutable, by which is meant incapable of revision and of variation.

This statement, in so far as it refers to judgments of beauty needs expansion. It is plain that logical intuitive judgments are immutable (the judgment, for instance, that "two things which are equal to a third thing are equal to one another" is clearly one that cannot be revised and cannot vary); but the immutable character of judgments of beauty may be doubly obscure. It may be obscured in the first place by the fact that our tastes develop. But when taste has developed we do not pass judgment upon a referend identical with that upon which our original judgment was passed; we pass a new judgment upon a new referend. For the referend of a judgment of beauty is not a block of marble in itself, or a series of sounds in itself, but it is the complex whole of which the block of marble, or the series of sounds, forms only one among other con-

stituents. The judgments passed when we see *The Soul's Awakening*, or read the verses of Adelaide Anne Procter, at the age of thirty-five are judgments passed on referends different from those on which we passed judgment when we saw *The Soul's Awakening*, or read the verses of Adelaide Anne Procter, at the age of fifteen, and it may be that at fifteen we fail when we see a drawing of Blake's, or read a poem of Wordsworth's, to pass a judgment of beauty; whereas, when we see the same drawing, or read the same poem, twenty years later we may pass such a judgment. Here once more the referends of our judgments are different.

Again, when a critic says to us, "How cool that water-color is!" we may find ourselves passing a judgment upon a new and beautiful referend, whereas before he spoke we passed judgment upon a referend that was merely dull.

In all these cases the different pairs of referends have, of course, elements which are common; the same pigments persist upon the same canvas, and the same printed marks upon the pages of the book. More than this, much of the imagery which contributes to the complex whole may be closely similar in the case of different referends.<sup>13</sup>

In the examples given above we are dealing with very different events. In the case of the fifteen-year-old girl and *The Soul's Awakening*, we have a positive judgment which claims to predicate beauty, whereas, in the case of the untrained observer and the "dull" water-color we have a negative judgment which denies beauty. In analyzing the important difference between these two examples we may discover the other factor which obscures the immutability of our judgments of beauty.

<sup>13</sup> This is true not only in the case of the referends of different judgments passed by the same person, but also in that of different referends upon which judgments are passed by different persons. Were this not so, we should be wholly indifferent to the beauty-judgments of other people. It would be impossible to be interested in a judgment passed upon a referend the nature of which was entirely unimaginable.



The second of the two cases is less complex than the first; it is one of failure adequately to apprehend a work of art. Once adequate apprehension is achieved the intuition of beauty follows; when "this" comes into being then the proposition "this is beautiful" is self-evident. Any spectator achieving adequate apprehension must pass the same judgment.

But suppose, *per impossibile*, that a spectator capable of predicating beauty in the case of, say, one of the most successful of Turner's water-colors were to apprehend *The Soul's Awakening* as the girl of fifteen apprehends it; would such an observer pass the judgment: "This is beautiful"? Surely he would not. How then is it possible to maintain the self-evident character of judgments of beauty? How can a judgment of beauty be described as immutable? It is plain that if the fifteen-year-old girl's judgment is an artistic intuition the position which this article sets out to maintain is wholly untenable. What is self-evident in the sense in which I use the term, cannot be self-evident to one person and not to another. If the intuitive character of aesthetic and of artistic judgments is to be asserted, it will be necessary to deny that the girl's judgment is an artistic judgment. And this I am prepared to do.

It may be objected that such a denial involves an arrogant assumption to the effect that: "My judgments on works of art are genuine artistic judgments; whereas those which are incompatible with mine are not." But such an objection would be ill-founded. To assert that there is a definable distinction between genuine artistic judgments and judgments which falsely claim to be artistic, is quite compatible with the admission that probably many, or indeed most, of one's own would-be artistic judgments are as a matter of fact not artistic at all. To distinguish in practice between genuine and spurious artistic judgments,

as between genuine and spurious works of art, may be an extremely difficult and subtle task; but it is the task of the art critic not of the would-be philosopher. In particularising to the extent of mentioning *The Soul's Awakening*, Adelaide Anne Procter, and Turner, I have been trespassing, perhaps illegitimately, upon his province. All that is incumbent upon me here is to establish the bare validity of the distinction and to show that failure to recognize its existence tends to obscure the immutable character of artistic judgments.

An attempt to analyze a little more fully the distinction between genuine and spurious artistic judgments will be made in the next section of this article. It is perhaps enough here to assert baldly that the girl admiring *The Soul's Awakening* is merely achieving the gratification of an instinctive, or libidinous, craving, and to add that *The Soul's Awakening* is a work of art only in Doctor Freud's sense of the term.<sup>14</sup>

Very little need be added to this statement. It is inserted

<sup>14</sup> "There is a path from phantasy back again to reality, and that is—art. The artist is one who is urged on by instinctive needs which are too clamorous; he longs to attain to honor, power, riches, fame and the love of women; but he lacks the means of achieving these gratifications. So, like any other with an unsatisfied longing, he turns away from reality and transfers all his interest, and all his Libido, too, on to the creation of his wishes in the life of phantasy. . . . The way back to reality is found by the artist thus: He is not the only one who has a life of phantasy; the intermediate world of phantasy is sanctioned by general human consent and every hungry soul looks to it for comfort and consolation. But to those that are not artists the gratification that can be drawn from the springs of phantasy is very limited. . . . A true artist has more at his disposal. First of all he understands how to elaborate his day-dreams so that they lose that personal note which grates upon strange ears and becomes enjoyable to others; he too knows how to modify them sufficiently so that their origin in prohibited sources is not easily detected. Further, he possesses the mysterious ability to mould his particular material until it expresses the ideas of his phantasy faithfully; and then he knows how to attach to this reflection of his phantasy-life so strong a stream of pleasure, that, for a time at least, the repressions are out-balanced and dispelled by it. When he can do all this, he opens out to others the way back to the comfort and consolation of their own unconscious sources of pleasure, and so reaps their gratitude and admiration; then he has won through his phantasy—what before he could only win in phantasy; honor, power, and the love of woman." *Introductory Lectures on Psycho-Analysis* (translated by Joan Riviere, p. 315)

At a general meeting of the British Psychological Society, held in November, 1923, Mr. Roger Fry criticized the above view of art in an interesting paper, since reprinted by the Hogarth Press.

merely to guard against any possible misunderstanding either of certain phrases in the preceding paragraph, or of the footnote No. 3 referring to "contact between minds." The point which it is intended to make clear is that even if the referend of two intuitive judgments is identical yet the judgments which refer to it are separate. It is certain that two minds may pass judgment on the same logical referend, and it is possible that two minds may pass judgment on the same aesthetic or artistic referend, but the acts of the two minds in judging are in both cases two and not one. Whether or not "joint enjoyment" is possible, "joint contemplation" (in Professor Alexander's sense of the term) is not possible.

## (ii)

We may now proceed to state two respects in which logical judgments and judgments of beauty differ from one another.

1. Logical intuitive judgments which are passed by different individuals may have the same content, whereas judgments of beauty passed by different individuals—whose minds are not in contact<sup>15</sup>—can never have the same content.

For instance, when A judges that "any two things which are equal to a third thing are equal to one another," he is passing an intuitive judgment having exactly the same content as the judgment to the same effect passed independently by B. The referend—namely, any two things which are equal to a third thing—is precisely the same in both cases. There is no need for joint enjoyment, for contact between the minds of A and B. A may be a Chinese mathematician and B an English schoolboy ignorant of A's

<sup>15</sup> See footnote No. 13.

existence. But when the similar judgments which A and B pass are aesthetic or artistic judgments, then the referend in each case is a complex whole among the constituents of which are included either the images or the emotional experiences of the person passing the judgment. The referends cannot be identical, unless we assume that the two minds making the two judgments are in contact, or have joint enjoyment, even to the extent of having certain of their images or of their emotions in common.

The referends of two aesthetic or artistic judgments—even when there is no contact—may, as it has already been suggested, be very similar; the point is that, however close their similarity, they cannot be identical.

2. We may now pass on to state the second point of difference between logical judgments and judgments of beauty, namely, all judgments of beauty are intuitive, whereas intuitive judgments form only a small proportion of logical judgments. This has already been stated by implication in Section I, and needs no further comment here.

## V

Omitting further discussion of the points of similarity and of difference between logical judgments and judgments of beauty, we will now turn to the final and perhaps the most controversial section of this article, namely that concerned with the predicate of an aesthetic or of an artistic judgment; in other words, we must consider what we mean when we say that something is beautiful.

I must admit that although I have used the term beautiful throughout this article as the predicate of both aesthetic and of artistic judgments, I have done so with very great hesitation. For some purposes it seems to me convenient

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to have a single term to cover the predicates of both classes of judgment. But on the other hand there are very serious drawbacks indeed to the use of a single term in two senses which differ so considerably. I have, however, sought in vain for terms which would be suitable substitutes for "beautiful, aesthetically" and "beautiful, artistically." There appears to me something to be said for using "beautiful," without qualification, for the former, and "artistically sound" for the latter. But "artistically sound" is a clumsy phrase; and "beautiful" is not wholly appropriate to every referend of an aesthetic judgment. There seems to be a decided need for new terminology. Meanwhile I am continuing to use "beautiful" for want of better words.

Let us now consider beauty as a characteristic of a complex whole having a natural object as one of its constituents. (Plainly, in passing appreciative judgments on such complex wholes it is not always beauty that we predicate of them. We may, without being unduly sensuous or unduly sentimental, pass judgments which introduce the word "beautiful" and yet are purely utilitarian, libidinous, or sentimental, and which would be more accurately stated in the form: "This is a luscious peach," "a desirable woman," "a rich harvest," "a delightful baby," or "a familiar and beloved glade."<sup>16</sup> Nothing more need be said of such judgments; but it may be repeated that the practical difficulty of distinguishing between them and our aesthetic judgments does not in the least invalidate the distinction between them.)

We pass a genuine aesthetic judgment only when we apprehend its referend as worth while apart from any utilitarian or libidinous considerations whatever. In other words, the value of the referend is not dependent upon any appetitive or emotional satisfaction we may derive

<sup>16</sup> Cf. Clive Bell: *Art*, p. 15.

from it; its value is intrinsic not instrumental. Aesthetic apprehension does not gratify desire but purges us of desire. As T. E. Hulme has it in one of his poems: "Loveliness is her own eunuch." Beauty is an indefinable characteristic of certain complex wholes which are apprehended as worth while in themselves. It is indefinable as yellow is indefinable; it is simple and ultimate; it is not possible and never will be possible to analyze it into constituents and show that their combination must give beauty.

In asserting that it characterizes certain complex wholes which are apprehended as worth while in themselves, I am not, of course, asserting that being worth while in itself is beauty. Sidney's face, as he gave to the dying soldier the water which he himself desired, may or may not have been observed by an onlooker, and may or may not have formed a constituent of a complex whole having beauty. But under no circumstances does it appear to me legitimate to predicate beauty of the complex whole which included as a constituent Sidney's *action* in giving the water. This complex whole may form the referend of an ethical but not of an aesthetic judgment. (It would, of course, have been no less worth while in itself had Sidney's face been distorted by his agony of thirst.) All things that are beautiful are worth while, or valuable, or ought to exist; but not all things that are worth while (or valuable, or which ought to exist), are beautiful. A judgment predicating beauty is a judgment making a unique and undefinable predication of certain complex wholes which are valuable. And since an aesthetic judgment involves the apprehension of something which is held to be valuable, it may be regarded as a value judgment when contrasted with a logical judgment, which is solely existential in character.

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We may now briefly consider beauty as a character of a complex whole among the constituents of which is a work of art. A work of art has already been defined, in Section III, as the adequate expression, in a medium, of an impression which has been apprehended by the artist as worth while in itself. In achieving such expression the artist produces something which may combine with certain of a spectator's images to form a complex whole possessing the indefinable characteristic beauty. It must be noted that the artist does not achieve his end by attempting to produce beauty, but by attempting successfully the expression of an impression.<sup>17</sup> A deliberate attempt to produce beauty produces a work of pseudo-art; it does not satisfy an artistic need but gratifies a libidinous craving.

A valuable experiment in psychological analysis may be provided by an attempt to classify the generality of Royal Academy exhibits<sup>18</sup> according to the mode in which they gratify libido, or instinctive need. Thus, pictures of young children, of mothers with babies, and of animals at play, may gratify the craving for object-love in its maternal form, while a more specifically sexual gratification may be secured by pictures of courting couples and by studies of the nude. Narcissism may be furthered, sometimes through self-identification with the royal, rich, complacent, or full-fed personages portrayed, sometimes through identification with the painter, as in the case of certain *genre* scenes and photographic landscapes, in which his dexterity is conspicuously manifested, and his patience in the face of difficulties.<sup>19</sup> The gratification provided by problem-pictures is more direct, and resembles that experienced upon the completion of a jig-saw puzzle; not dissimilar is the gratification which accompanies the recognition of an ex-

<sup>17</sup> Cf. Lascelles Abercrombie: *Towards a Theory of Art*, p. 59.

<sup>18</sup> Cf. Roger Fry: *The Artist and Psycho-Analysis*, p. 20.

<sup>19</sup> The lady exclaiming, before *The Derby Day*, "Excuse me, Sir, but can you tell me if this was really done by hand?" was probably experiencing vicarious satisfaction in Frith's achievement.



hibit as the latest of a familiar series; the spectator, upon seeing a flock of sheep in a snowy lane, or a group of ruddy boy-bathers in the act of discarding their shirts, is able to identify the painter and name him confidently to a companion. Further examples of this method of analysis are perhaps unnecessary.

It need scarcely be added that it is not the choice of a subject which makes many of the Royal Academy exhibits suitable objects for psychological rather than for artistic consideration. It is clear that studies of the nude, and, even, portraits of royal personages, may be works of art. Nor is it suggested that works of art never appear at Burlington House. (Again, it must be pointed out that the terms artistic and pseudo-artistic are not exhaustively descriptive of Academy, or other, pictures. Many of the works at Burlington House are capable of being apprehended aesthetically even when they are not works of art. Such pictures fall into the category I have described as reproductive-aesthetic construction.)

The critic of the view of art here suggested may object that the distinction drawn between genuine and pseudo art is a false one, and that the difference is of degree, not of kind. "It is merely a question as to the complexity of the disguise assumed by the gratified libido. You describe *The Soul's Awakening* as a work of pseudo-art because you are now aware that it once gratified your libido; you describe a Turner water-color as a genuine work of art because you are unaware that it now gratifies your libido. The difference consists simply in complexity of disguise, and upon your ability to recognize certain facts and your failure to recognize others. The argument breaks down completely.

To deal adequately with this criticism, which is, I think, very plausible and very unsound, would take more space than is here available. It must suffice to refer the critic

to Freud's *Beyond the Pleasure Principle*, and to suggest that art may have a very close connection with that tendency which Freud believes to be in sharp contrast to libido, with the tendency, namely, to turn from the dynamic to the static, from "life" to "death," adding dogmatically that whereas the apprehension of and the construction of pseudo-art gratifies unconscious desire, and whereas aesthetic apprehension purges us of desire, desire is simply irrelevant in the case of the apprehension of, and the creation of, genuine art. Art is not justly described as a means of evading reality; rather is it an attempt to achieve reality of another kind than that of the vital world of pain and pleasure.<sup>20</sup> The temptation to elaborate this theme at the close of an article which is already long must be resisted.

How the artist sets about the work of expressing his impression, and, if he succeeds, creates a work of art and achieve beauty; how the spectator, or the audience, adequately responds to his achievement; how and why artist or audience may fail; these too are provocative questions which lie outside the scope of this article, and must, for the present, be deferred.

SUSAN MILES.

LONDON, ENGLAND.

<sup>20</sup> Cf. T. E. Hulme: *Speculations*. "Modern Art and Its Philosophy."

## THE ORIGINAL NATURE OF MAN IN EARLY CHINESE SPECULATION

### HISTORICAL INTRODUCTION

SOCIOLOGISTS and historians of social thought have neglected the contributions of the Chinese. I have thought to remedy this neglect in a measure by summarizing particular contributions by typical Chinese thinkers on various phases of social and economic speculation. Some small steps in this direction were taken in a previous writing.<sup>1</sup> Here I shall confine myself to several lines of thought developed in China during the period of China's greatest intellectual efflorescence—the time of the philosopher Meng (Mencius). Meng was China's greatest preacher of the doctrines of Kung fu tze (Confucius).<sup>2</sup> and his teachings, with those of his master, represent to this day the substance of orthodox Confucianism. It was Meng who saved the day for Confucianism of the orthodox against the onslaughts of an host of heterodox Confucianists and Taoists.

Speculations on the original nature of man are especially rife in the Confucianist discussions of Meng's day. Meng propounded the theses which are now considered orthodox and which were essentially those of his master. These theses were clarified in the intellectual war result-

<sup>1</sup> MacLeod, *The Origins of the State*, Phila., 1924, p. 43, etc.

<sup>2</sup> The Chinese term *tze*, Latinized becomes *-cius*. Therefore Kung fu tze becomes Confucius; and Meng tze becomes Mencius. I personally think it not desirable to Latinize in English usage any Chinese names unless we Latinize them all. The English tongue is nowadays becoming used to Oriental names.

ing from the contrary theses of unorthodox followers of Kung. We have no adequate record of the sayings of the opponents of Meng, but we have very adequate record of those of disciples of theirs who lived shortly after Meng and whom we may take as representative of the unorthodox Confucianist thought of Meng's years.

Meng lived between 371-288 B. C. His relationship to Kung might be compared to that of Plato to Socrates. Meng began his teaching while Plato was still living, but Kung died one hundred and eighty years before Meng's birth, and during this nearly two centuries from which no writings of followers of Kung's remain, Kung's ideas were echoed by at least his grandson, Tsze-sze, and persisted in vitality under the attacks of Taoists and hedonists and utilitarians such as Kwang, Mu, and Yang, until the supreme champion Meng appeared, who faced these old enemies and also the new ones of his own generation.

This efflorescence of Chinese thought lasted till 221 B. C. It had been flowering from about 800 B. C. through the days of Kung (circa 500 B. C.), blossoming in the days of Meng (circa 325 B. C.); and its waning was indicated some time before 221 B. C. in the development of a school of sophistry led by Ching Sun Ling and his followers. This period of nearly six hundred years was characterized by the division of China into a multitude of petty kingdoms, each warring against the other; each tending to grow permanently by absorption of the territory of neighbors. In 221 B. C., for the first time in history, the small kingdoms and dukedoms of feudal China were welded into a vast empire. This work was the result of the violent conquests of Shi Huang Ti—unquestionably the greatest state-builder of all times and races.

This conqueror founded the Ch'in dynasty of unified China. As a conqueror, he had much to suppress, and growing in time more and more bitter at his critics of all

kinds, he put an end to philosophical development, ending, perhaps, only the decay which the rise of the sophists may be held to represent. By 213 B. C., Shi Huang Ti had buried alive all who dared voice a criticism of his empirical authority, and had burned every book on speculations of any kind which he could find, except the ancient *Book of Changes* which he could not understand and which he concluded was merely a book of divination, and harmless so far as his tyranny was concerned.

The lethargy consequent upon the suppression under the reign of this first emperor of the Ch'in dynasty persisted all through the centuries of the supremacy of this dynasty—until its end in 959 A. D. By this latter date the thinkers of India were beginning to influence Chinese thought, and stimulate it anew, consequent upon the introduction of Buddhism into China.

In the thinkers of the pre-Ch'in period of Chinese thought whom we consider in this paper we are dealing only with those whose speculative developments are out of an indigenous or native Chinese background. The thought of Kung, and his great protagonist Meng—the thought development we know of as orthodox Confucianism—is peculiarly native to northern China. Both Kung and Meng, moreover, were born, and lived nearly all the time, in Shantung. The heterodox Confucianists we shall consider very possibly were of a south Chinese influence, but still they remain distinctly Confucianist in opposition to the Taoists.

In the present state of historical investigation one can not feel too ready to draw conclusions, but there is little doubt that the monistic speculations of Lao tze, father of Chinese Taoism—represented in aberrant developments by such as Yang the hedonist—is the product of non-Chinese intellectual developments. It is probably the result of influences which, sometime before 500 B. C., entered

China from the southwest, and may be traced back to Indian influence. It is significant that Lao tze himself was a native of an outlying or border state through which exotic culture influences were moving. The introduction of Buddhism before the end of the Ch'in dynasty followed by a second stimulation of Chinese thought such as may perhaps be attributed to the introduction of Indian influences in the form of the monism which gave rise to Taoism — was but a second great movement which probably followed the same routes as the earlier one in reaching the Chinese.<sup>3</sup>

The contemporaneous development of intellectual efflorescence in China and in Greece, partly because of many unique parallellisms in Chinese and Greek thought, might lead one even to go further and suspect a transmission of influences between the ancient Mediterranean and ancient China. A study of ancient trade connections lends at least an element of possibility to this supposition.<sup>4</sup>

## II

### MENG AND THE INNATE GOODNESS OF MAN

In contrast to the monism of the Taoists, Confucianist thought was pervaded with the concept of universal dualism. This dualism appears in speculations upon the origi-

<sup>3</sup> Nestorian Christianity entered China in 636 A.D. Judaism entered probably somewhat earlier.

<sup>4</sup> On the common origins of Chinese and Babylonian culture, and the diffusion of culture from India to China consult B. Laufer: *Some Fundamental Ideas of Chinese Culture*, Journal of International Relations, v. 5, 1914-1915; and C. W. Bishop: *The Geographical Factor in the Development of Chinese Civilization*, The Geographic Review, 1922.

On the early history of China one might consult F. Hirth, *The Ancient History of China to the End of the Ch'ou Dynasty*, 1911.

The metaphysics of the early Chinese is covered by D. T. Suzuki: *A Brief History of Early Chinese Philosophy*, 1914.

The history of Meng's period is covered in the introduction to Legge's translation of Mencius' work and that of some of his contemporaries and followers; see J. Legge: *The Life and Works of Mencius*, 1895.

nal nature of man, in the doctrine that in every human breast there are both egoism and sympathy. The controversy between Meng and the heterodox was as to the relative dominance of the one or the other—the impulse to good or to bad.

Meng insisted that innately every human being is good; that is, that he is born with a predisposition to sympathy which altogether overshadows all impulses which might be called collectively—egoism. Experience may sometimes lead to a weakening of sympathy and a development of the latent egoism; but such a rising of egoistic impulses with the consequent evil or unsocial behavior, is the result of unfortunate environment and not evidence that the resultant egoistic individual has been born with any dominance of egoistic impulses. Man is not like the animals in whom, innately, egoism dominates; sympathy is by nature ascendent in his breast. Pointing out this fact that the original nature may deteriorate during the process of living and insisting on this distinction between men and the lower animals Meng said, concerning human sympathy:

But there is some growth of its life day and night; and in the air of the morning, just between night and day, the mind feels in a degree the desires and aversions which are proper to humanity; but the feeling is not strong, and then it is fettered and destroyed by what the man does during the day. This fettering takes place again and again; the restorative influence of the night is not sufficient to preserve (the natural goodness of the mind); and when this proves insufficient for that purpose, the nature becomes not much different from that of the irrational animals, and when people see this they think that it never had these powers. But does this condition represent the feelings (natural) to humanity?<sup>5</sup>

<sup>5</sup> Book 6, Part 1, Chapter 8, Sections 1, 2.

*Jen* is translated sympathy in the sense of fellow-feeling. It is essentially the sympathy of Adam Smith and the Consciousness of Kind of Giddings.



Meng, following Kung—and all Confucianists since have so taught—taught that sympathy, the ascendant principle in man's nature, is the socializing principle. It is the motive force behind social living and the origins of forms of social behavior. Sympathy is the *Tao*—the road, the door—of social living.<sup>6</sup> Mencius said:

Everybody has a feeling for others which he is unable to endure. Suppose a child is at the point of slipping down into a pit. It awakens in the spectator a mingled feeling of apprehension and compassion, which urges him to an immediate rescue of the child. This is not because he wants to incur a favor upon its parents. This is not because he wants to be honored by his friends or fellow-villagers. This is simply because he can not bear its pitiful scream. Men who have no feeling of pity are therefore not human beings.<sup>7</sup>

If it were not for the dominance in man of this sympathy over egoism or self-interest, society would fall to pieces and every man would be at every other man's throat.

It is the task of men who wish to follow the *tao* to resist the corruption of their original nature, and:

The great man is he who does not lose his child's-heart.<sup>8</sup>

The traditional founders of Chinese social organization—the Sage Kings—attained the perfection of character by perpetuating throughout the experiences of life the preponderance or dominance of sympathy with which nature endows the heart of a child. Their perfection is within the reach of all. By guiding our behavior according to

<sup>6</sup> In contradistinction to the more mystical *Tao* of the followers of Lao tze, eventually called Taoists.

<sup>7</sup> Suzuki, p. 53, cites Han-fei-tze's definition of *jen* or sympathy, which we may quote. "*Jen* is to love others with gladness of heart; to rejoice when they are blissful; and to be grieved when they suffer misery. This is because the heart is unable to refrain from being so affected, and has nothing to do with a desire for compensation." He adds: "Superior *jen* works as if not working."

<sup>8</sup> Book 4, Pt. 2, Sec. 12.

the rules or propriety formulated by them, we can avoid experiences which will nurture egoism or self-interest.

Wear the clothes of Yaou; repeat the words of Yaou; do the actions of Yaou—and you will be a Yaou.<sup>9</sup>

All men are born with the same natural endowments; we vary merely in the nurture of them:

The sage and we are the same in kind.<sup>10</sup>

If it receive its proper nourishment there is nothing which will not grow; if it lose its proper nourishment there is nothing which will not decay away.<sup>11</sup>

So, of the virtues which are the product of sympathy:

Hence it is said—"Seek and you shall find them; neglect, and you shall lose them."<sup>12</sup>

Meng troubled himself little with explaining how culture or social organization arose. He did, however, plainly conceive that institutions were *not* fetters on human self-expression; they were merely ways in which sympathy could find smooth channels where irritations and irregularities would give no cause to the upspringing of self-interest. On the origin of the elaborate mortuary rites of the Chinese Meng observed that originally the bodies of parents were merely thrown into ditches, to be eaten by foxes and wildcats, flies and gnats; but in time:

The emotions of people's hearts affected their faces and eyes, and they then covered the bodies of parents with earth.<sup>13</sup>

<sup>9</sup> Book 6, Pt. 2, Chap. 2, Secs. 1, 4, 5.

<sup>10</sup> Book 6, Pt. 1, Chap. 6, Sec. 7. Cf. Book 6, Pt. 1, Chap. 15.

<sup>11</sup> Book 6, Pt. 1, Chap. 8.

<sup>12</sup> Book 6, Pt. 1, Chap. 6, Sec. 7.

<sup>13</sup> Book 3, Pt. 1, Chap. 5. In this conversation with E Che—a disciple of Mu's—Mencius is protesting against the utilitarian objections of Mu's to the expensive elaborateness of Chinese mortuary customs. Mencius of course had no actual knowledge of how the prehistoric Chinese disposed of their dead. Many surrounding Mongoloid peoples however did and do practice exposure of the dead, and of this Mencius undoubtedly knew.

Mencius, it is apparent, begins with the presumption that the Chinese culture of his day represented the perfection of a long development of forms of human behavior which had arisen from the desire of men's hearts to express their innate tender emotions. Therefore, all feelings which manifest themselves in the face of problems of behavior are merely phases of sympathy in its various linkages with the will, intelligence, and bodily desires and impulses. The circumstances under which we live and move are ever changing, and our feelings respond to the varying problems as to what is the proper behavior. Sympathy then finds itself variously differentiated under the names of the virtues—loyalty, filial piety, courage, propriety, faithfulness, righteousness, long-suffering, benevolence. Meng writes, for example:

The feeling of compassionate distress belongs to all men; so does that of shame and dislike; that of modesty and respect; and that of approving and disapproving. The feeling of compassion and distress is the principle of benevolence; the feeling of shame and dislike is the principle of righteousness; the feeling of modesty and respect is the principle of propriety; and the feeling of approving and disapproving is the principle of knowledge. Benevolence, righteousness, propriety, and knowledge are not fused into us from without; they naturally belong to us.<sup>14</sup>

All these virtues men are therefore innately predisposed to because "the nature of man is good"; in it sympathy is the dominant note. Of course, if through unfortunate environment and experience self-interest or egoism becomes uppermost in anyone, then these virtues would be replaced by their opposites, the vices, which are the manifestations of egoism in the face of behavior problems.

These assertions of Meng's were made in the face, as we have noted, of vociferous denial by the heterodox Con-

<sup>14</sup> Book 6, Pt. 1, Chap. 6.

fucianists, among other schools of thinkers, of his day. The writings of these contemporaries have not come down to us, but some notes of their sayings we do have, and it is evident that their thought is represented by the two writers whom we shall now consider.

### III

#### SEUN K'ING, AND THE INNATE BADNESS OF MAN

Seun, who was in office under King Seang of Ts'e from 271-264 B. C., was probably beginning his teaching during the last years of Mencius' life. He brilliantly criticised Meng's ideas of the innate goodness of man. He maintained that human nature was essentially evil—that is, that egoism was innately dominant over sympathy. Naturally therefore, Seun's interpretation of the nature and origin of human institutions would be different from that of Mencius. It was Seun who exclaimed:

Man, as he is born, has in him nothing but the elements of disorder, passive and active.

Without the artificial restraints of law, the strong would be injuring the weak and spoiling them; the many would be tyrannizing over the few and looting them; a universal disorder and mutual destruction would speedily ensue.

It thus appears that the following of man's nature and yielding obedience to its feelings will assuredly conduct to contentions and robberies, to the violation of duties belonging to everyone's lot, and to the confounding of all distinctions, till the issue be in a state of savageism.

Everyone has inborn desires. When these desires are not satisfied, he looks around for the objects of satisfaction. When no measure and limits are set to this searching, there necessarily arises quarreling.<sup>15</sup>

<sup>15</sup> Translation of Seun's *Essay* in Legge, op. cit., pp. 104, 77-78.

During Seun's life, this same thesis was perhaps equally ably maintained by the philosopher Hsün.

The natural impulses of man make for disorder and violence, quarreling and misery. Therefore:

There must be the influence of teachers and laws, and the guidance of propriety and righteousness, from which will spring self-denial, yielding to others, and an observance of the well-ordered regulations of conduct, till the issue will be in a state of good government.

Under institutions of social control, man is made to act as if his nature was good. But "The nature of man is evil; the goods which it shows is factitious."<sup>16</sup> He argues that the very existence of institutions of social control are evidence of the innate badness of man, for:

If man's nature be really so correct, regulated, and happily governed in itself, where would be the use for the Sage Kings, propriety, and righteousness? What could they add to the nature so correct, regulated, and happily ruled in itself.<sup>17</sup>

He insists that although it is a fact that in society men appear to be yearning for order and decency, and to dread violence and disorder, yet this too is merely evidence of man's innate badness, particularly inasmuch as even with this desire for order present among men, it remains difficult for men to attain a state of order, or to maintain that which is attained:

Again, the fact that men wish to do what is good, is because their nature is bad. . . . It is a fact indeed, that by

<sup>16</sup> *Ibid.*, pp. 78 seq. Seun ends his great essay with the exclamation: "All is the influence of association. All is the influence of association!"

By way of definition, in this connection, Seun writes: "In ancient times and now, throughout the empire, what is meant by good is a condition of correctness, regulation, and happy government; and, by evil,—a condition of deflection, insecurity, and refusing to be under good government. In this lies the distinction of good and evil."

<sup>17</sup> In Hsün's words also: "human nature is but the original foundation and raw material; artificiality means refinement and culture. If not for the evil and rudeness of the original nature, artificiality would have nothing to apply itself to. (*Ibid.*, pp. 105-106.)

nature man does not know propriety and righteousness; he therefore thinks and reflects and seeks to know them. It is a fact that he does not know propriety and benevolence; therefore he studies them with vigorous effort and seeks to have them. . . . (The thin wishes to be thick; the ugly wishes to be beautiful; the narrow wishes to be wide. The poor wish to be rich; the mean wish to be noble.) When anything is not possessed in one's self, he seeks for it outside himself. (But the rich do not wish for wealth; the noble do not wish for position.) When anything is possessed by one's self, he does not need to go beyond himself for it.

If we speak of the fondness of the eyes for beauty, or of the mouth for flavors, or of the mind for gain, or of the bones and flesh for the enjoyment of ease; all these grow out of the natural feelings of man. The object is presented and the desire is felt; there needs no effort to produce it. *But*, when the object is presented and the affection does not move till after hard effort, I say that this effect is factitious.

Meng said, "Man has only to learn, and his nature becomes good," but I reply, It is not so. To say so shows that Meng has not . . . examined into the difference between what is natural in man and what is factitious.

The natural is what the constitution spontaneously moves to. It needs not to be learned. It needs not to be followed hard after. Propriety and righteousness are what the Sages have given birth to. These, men become capable of only by learning; men achieve them only by hard practice.

That which is in man, not needing to be learned or striven after is what I call natural; that in man which is attained to by learning, and achieved by hard striving, is what I call factitious.<sup>18</sup>

Seun conceives of institutions of social control as having the function of restraining, checking, fettering the natural man, and preventing the disorder which would otherwise result in the face of the fact that while the desires of

<sup>18</sup> *Ibid.*, pp. 78, seq.

men are insatiable the objects of their satisfaction are limited:

Meng says, "The nature of man is good, but all lose and ruin this nature, and therefore it becomes bad." But I say that this representation is erroneous. Man being born with his nature, when he thereafter departs from its simple constituent elements he must lose it. From this consideration we may see clearly that man's nature is evil. What might be called the nature's being good would be if there were no departing from its simplicity to beautify it; no departing from its elementary dispositions to sharpen it. . . .

It is the nature of man, when hungry to desire to be filled; when tired to desire to rest. These are the feelings and nature of man. But now—a man is hungry, and in the presence of an elder he does not dare to be the first to eat; he is yielding to that elder. He is tired with labor, yet he dare not ask for rest; he is working for some one. . . .

It appears that if there were an accordance with the feelings and the nature, there would be no self-denial and yielding to others. Self-denial and yielding to others are contrary to the feelings and the nature.

Social organization—or, in other words, institutions, ordered behavior, social control—he conceived of as having been developed through the exertions of exceptional men—the traditional Sage Kings of Chinese history. Of the origin of Kings, first of all, he has this to say:

The rise of princes and governors . . . is to be traced to the badness of human nature (just as the origin of the pressing boards is to be found in the crookedness of the wood, and the origin of the carpenter's marking line is to be found in the fact that things are not straight).

He implies that originally there obtained a "state of war," that wicked kings were rising and falling; that in time there appeared the Sage Kings, who put an end to the state of war by establishing rules, or social order, so that



no longer would everyman's hand be against every other man.

The Sage Kings did not differ from other men in the evil of their original nature. They merely were possessed with a greater desire for peace, along with power over others, which led them to evolve schemes for social order which would gratify the desire of evil men to live in a way in which there would be peace. We may ourselves attain perfection such as that of the sage kings only by behaving like the sage kings; by strict obedience to the laws and customs which are designed to check our innate egoism:

The man who is transformed by teachers and laws and proceeds in the path of propriety and righteousness—is a mean man,<sup>19</sup> that is, of the middle type.

Difference of social environment, he indicates, account for the differences of character among men.

Hsün, also insisting on the same points here made by Suen, is a little more lucid in this matter:

Moral training cannot gain a step by mere introspection. Let a man studiously apply himself, instead, to discipline.

One whole day spent in meditations does not equal in merit one minute of study. (We may stand on tiptoe as far as possible, but much better to climb some elevation.)

Wise men do not differ in their nature from others. What makes them wise is due to their adaptation to cultural environment. Therefore wise men are particular in choosing their place of dwelling and their associates.<sup>20</sup>

Of the institution of social order by the Sage Kings, Suen has this to say. The Kings regretted that man was living in anarchy and evil, and to control man's evil nature:

Set up therefore the majesty of princes and governors to awe it; and set forth propriety and righteousness to

<sup>19</sup> *Ibid.*, pp. 78 seq.

<sup>20</sup> *Ibid.*, p. 108.

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change it; and framed laws and statutes of correctness to rule it; and devised severe punishments to restrain it. So that its expression might be under the dominion of rule, and in accordance with what is good.

An inquirer will ask, "If man's nature be evil, whence do propriety and righteousness arise?" I reply, All propriety and righteousness are the artificial production of the sages and are not to be considered as growing out of the nature of man. . . . The sages pondered long in thought and gave themselves to practice, and so they succeeded in producing propriety and righteousness. Thus it is that propriety and righteousness, laws and regulations, are the artificial product of the sages, and are not to be considered as growing naturally from the nature of man.

Quarreling means disturbance and disturbance means obstruction. Wise men of old hated this disturbance. Therefore they established rules of propriety and justice, and imposed them upon the people, whose desires were thus regulated and whose requirements were thus furnished. Every desire was not allowed to be satisfied, nor every satisfaction to lead to a new one; the equilibrium between them was kept under control. This is the beginning of the proprieties. . . .<sup>21</sup>

In arguing on this point Seun gives his argument a curious scholastic twist in order to win over his conservative hearers. The belief in the existence and transcendent greatness of the traditional, semi-mythical or mythical Sage Kings of Chinese traditional history was an article of faith with Chinamen in Seun's day. So Seun points out that

To say that human nature is good does away with the Sage Kings, and makes an end of propriety and righteousness. To say that the nature is bad exalts the Sage Kings, and dignifies propriety and righteousness.

In other words, a social philosophy of anarchism must result from Meng's teaching; from his own must result a

<sup>21</sup> *Ibid.*, pp. 78, *seq.*, and p. 104.

social philosophy which exalts the state and "law and order."

Suen himself proposed, and answered, a possible objection in connection with his proposition concerning the work of the sages.

An inquirer may say again, "Propriety and righteousness, though seen in an accumulation of factitious deeds, do yet belong to the nature of man, and because of this it was that the sages were able to produce them.

Unfortunately his only answer to this was to draw an analogy and point out that although a potter makes a design on a pot, the design is the creation of the potter and no part in its creation is to be attributed to the clay."<sup>22</sup>

#### IV

##### HAN WAN-KUNG AND THE ANIMALITY OF MEN

Han Wan-Kung, like Suen and Hsun was posterior to Meng but was merely following out a line of thought developed at least as early as the days of Meng's activity. He seems to have been a disciple of the heterodox Kaou of Meng's day. For we read in a conversation between Meng and his disciples the statement of a disciple:

The disciple Kung-too said, "Kaou says that Man's nature is neither good nor bad. Some others say that man's nature may be made to do good and it may be made to do bad. Others say that the original nature of some is good, of others, bad.

<sup>22</sup> Such appeals served to win over many Chinese thinkers in the course of the centuries to the doctrine of the evil of human nature. But Han-Yu was influential (during the T'ang dynasty) in making Meng's teachings "orthodox" and making it discreditable to anyone longer to accept Suen and Hsün's teachings. (768-824 A. D.)

In another conversation, Meng is informed that Kaou and others are maintaining that institutions are not, as Meng insists, the outgrowth or expression of human sympathy. That there must be a distinction drawn between "propriety" and "benevolence"; between, that is, natural goodness and orderly behavior. Benevolence or natural goodness was indeed prompted by impulse; it consisted in doing good things—such as loving a younger brother—not required by custom or law. Proper behavior however, such as deference in all things to an elder brother might have nothing to do with human impulse—or might be contrary to the desires of the younger person concerned; it may therefore be something forced upon a person "from without."<sup>28</sup>

According to Han, men are, according to their innate tendencies, exceedingly complex and of infinite variability. He conceives of three general grades or categories—the good, bad, and indifferent. In life, the original nature of a man may be gauged according to the way it expresses itself under categories of behavior which on their best side may be classified under the five headings—benevolence, righteousness, propriety, sincerity, and learning. It seems that to Han, of these five expressions of original nature, that benevolence is the keystone; that upon the possession or lack of a tendency towards benevolence depends the quality of a man's behavior.

Han states that men who are by original nature wholly good will rise above environment and lead a good life against all odds; men who are wholly bad by original nature can be coerced by law and custom into leading a fairly good life.

Han made a rather careful analysis of the emotional life of man. The feelings or emotions he classifies as seven: joy, anger, sorrow, fear, love, hatred, and desire. Men

<sup>28</sup> Legge's *Mencius*, Book 6, Pt. 1, Chaps. 4, 6. (In Mi's writings Kaou is called Shing.)

both good and bad have all of these. Perfection does not consist in the dominance or absence of any, nor in the weakness or strength of any. Men should hate as well as love. Perfection consists in emotional strength coupled with *emotional balance*. People weak emotionally are defective; people whose emotions are disordered—such as those who are always in a state of anger, incapable of hate or love, and so forth, are defective.

To Han, the emotional strength and balance of a person is a phase of original nature; it is innate and cannot be changed..

In controversy Han reasoned:

Speaking of the nature, Meng said, "Man's nature is good." The Philosopher Seun said: "Man's nature is bad." The Philosopher Yang said: "In the nature of man good and evil are mixed together." Now to say that original nature, good at first, subsequently becomes bad; or that bad at first, it subsequently becomes good; or that, mixed at first, it subsequently becomes either wholly good or wholly bad, is error. . . . In each of these cases only the nature of the middle (or indifferent) type or grade is dealt with, and the superior and inferior grades are neglected. Those philosophers who are right about one grade are wrong about the other two.

Han was nearer to Suen than to Meng in his philosophy. He was convinced that original nature could not be changed by environment; that goodness and badness in behavior is consequent merely upon goodness and badness in original nature of the individual. Original nature persisted through all environments. Persons born with intelligence and tendencies to kindness or benevolence would rise superior to an environment of evil; persons of bad natures by birth would fall from a good environment and would never rise from an evil one save by being lifted by force through the agency of institutional control. "Through the awe of power they may come to have few

faults" of behavior, writes Han. These opinions he sustains excellently by historical examples which illustrate his various points.<sup>24</sup>

## V

## THE GOLDEN AGE

We shall turn from our self-limitation in this paper in which we consider only Confucianist philosophers, to make note of the existence of the Golden Age concept among the Chinese of Meng's time. We have to go to a Taoist to find explicit the concept of the Golden Age, and we would like to make note of it in this paper inasmuch as we have a mind to compare the development of Chinese and of Greek social thought of this early period. It is Yang the hedonist who spoke of a Golden Age in the past. Now, his concept appears not to have been a result of deductions arising from observation of historical or psychological data, nor yet a survival of old myths. Yang Choo in this as in much else of his writing seems to be merely the poet and hedonist weary of the restrictions of Chinese law and order and imagining that men could not always have lived so; that institutions of restraint must have come to spoil an earlier existence of emotional calm, peace, certitude and quiet. So he writes:

People of olden times were perfectly aware of the shortness of life and wanted to make the best of it. They lived as their simple, innocent impulses dictated. Their desire was to preserve the naivete or integrity of their nature. They never worried themselves about things earthly. They

<sup>24</sup> Legge translates Han's work *An Examination of the Nature of Man*, in his *Life and Works of Mencius*, pp. 88, seq. The essay is taken from the collections of Han You, Chap. 1, Sec. 4, Part 3.

never distorted or mutilated what they obtained from heaven merely for the acquisition of things artificial. They were above political intrigues, aspiration for fame, commercial greed, and other petty human concerns.<sup>25</sup>

### CONCLUSION

It is evident that Meng conceived of man as always having lived in a state of harmony and peace, although he makes no positive note of a Golden Age. From this state they evolved progressively richer and more perfect ceremonialization of social intercourse making for a smoother and easier working of social life which would inhibit whatever tendencies in social living which might develop the modicum of self-interest or egoism which all men are by nature endowed with. Chinese social organization was an ideal rule of behavior which provided for the fullest expression of sympathy in human relationships and the fullest inhibiting of self-interest. Conformity to the Chinese pattern of behavior would lead one through a perfect life, full of human sympathy. Social organization was a gradual growth implicit in human nature—in which sympathy was the dominant note. The social order is the working out of sympathy seeking orderly and kindly modes of behavior.

The actual expression of a concept of the Golden Age awaited the poetizing of a Taoist Hedonist who was wholly out of sympathy with Meng's belief in the beneficence of institutional life.

In the heretical Confucianists Seun and Hsün, sympathy was denied as an attribute of the original nature of

<sup>25</sup> From *The Saying of Yang Choo*, in Legge's *Mencius*.



man. Men were possessed by nature of desires; inasmuch as the material conditions of existence were such that all the desires of all men could not possibly be satisfied, men originally lived in a State of War. Men, conscious of their misery longed for a way out, which was furnished by the evolution of institutions under the strong-man rule of the sage kings. Institutions accomplish an orderly distribution of the objects of the satisfaction of desires among men, and prevent the outbreak anew of a state of war in which a disordered and violent scramble for these objects results in alternating anarchy and tyranny.

The line of thought developed by Han insisted on the variability of human natures. Institutions were necessary only for the restraint of evil men.

WILLIAM CHRISTIE MACLEOD.

WHARTON SCHOOL, UNIVERSITY OF PENNSYLVANIA.

## THE PHYSICAL SCIENCE OF LEONARDO DA VINCI: A SURVEY

### I. INTRODUCTORY NOTE. THE RENAISSANCE IN SCIENCE

**I**T is the purpose of this article to discuss the character of the investigations of Leonardo da Vinci in the general field of cosmography, physics and mechanics. It will be desirable at the outset to consider briefly the scientific atmosphere of his time. Leonardo lived at the height of the Italian Renaissance. He was, in fact, its most vigorous and outstanding personality. So much has been written of this wonderful period in the intellectual history of mankind that we will do no more here than remind the reader that of the three factors in cultural progress—letters, art and science, the third was, in fact, the last to be caught up in the rejuvenating stream of the Renaissance. Let us briefly consider some of the factors which aided in the revolution in scientific outlook that was beginning to manifest itself in Leonardo's day. Broadly speaking the general legacy of the science to which the fifteenth century was heir was the mediaeval acceptance of the doctrine of macrocosm and microcosm as derived from Plato's *Timaeus*, a doctrine which had received added strength from the neo-Platonic vogue of the time. Again the so-called Aristotelian scheme of the four elements of earth, air, fire and water, in conjunction with the four primary qualities of hot, cold, moist and dry was the typical outlook in what we may perhaps call the world of chemistry, although the

alchemists were also by now developing their well-known theories of a salt mercury sulphur scheme. Again in cosmography the prevailing influence was still the geocentric system of Ptolemy.

Generally speaking, it may be said of a mediaeval learning prior to the Renaissance in Science that it was characterized by a subservience of all thought to the theological purpose, although a compromise had been effected between the Church and classical teaching. The outlook as a whole was circumscribed. The astrological implications of the doctrine of macrocosm and microcosm were sufficiently complete to be satisfying, and so no attempt was made to go outside it. But now, at last, a spirit of unrest was beginning to manifest itself. The word was suddenly coming to be opened up. The Portuguese navigators were evidencing not only a spirit of adventure which was in itself contagious, but were also providing new avenues of trade. As a result there was greater commercial prosperity. Thus people had more leisure for thought, a leisure, moreover, that was being fostered by the era of printing that had just begun. Again in the intellectual world the physical speculations of Albert of Saxony and of Nicholas of Cusa, the geological investigations of Palissy, and the new and reliable versions of Ptolemy's *Almagest* that had been made available by Peurbach and Regiomontanus, were all beginning to have their effect. The factors in the scientific Renaissance are not however complete without reference to the stimulus received in the direction of improved technological processes. These resulted not alone from the commercial prosperity of the times, but also from the stern necessities of an ever-present warfare. So we get the epoch-making work of Geo Agricola on the processes of metallurgy, and later the writing of Ramelli and others in mechanical technology. All these considerations came into operation at a time coincident with the conflict be-

tween the Protestants and the Church of Rome, and brought into being that great impetus in scientific outlook which led so magnificently to the achievements of such men as Copernicus, Kepler, Galileo and later Newton. Where in this scheme of things comes the subject of our study, Leonardo da Vinci? It is difficult to say. Leonardo was not a teacher in the ordinary sense. Living at a period when publication was possible, he published nothing. Yet always he wrote and wrote, and his accumulation of manuscripts teems with discussions of scientific problems covering an amazingly wide range. They show all too clearly how far in advance of his time was this great thinker, and how much the world has lost through the tragic dispersal of the notebooks shortly after da Vinci's death. Now that they have come to light, we are able to realize what a mighty factor in scientific evolution Leonardo might have proved.

## II. BIOGRAPHICAL NOTE IN RELATION TO SCIENCE

Leonardo da Vinci was born in 1452. He was the illegitimate son of Ser Piero da Vinci, a notary, and was brought up in his father's household. At the age of fourteen he was apprenticed to the famous Florentine artist, Andrea Verrocchio. We shall see that from this time onwards Leonardo was constantly moving in scientific circles. Verrocchio himself was a man of wide interests, which included a love of both geometry and perspective, and this naturally brought him into contact with all his contemporaries of a like calibre. From all of these da Vinci must have learnt. They included Giovanni Agiropulo the Aristotelian, Leon Battista Alberti, and Paolo Toscanelli the geographer. Consequently when Leonardo left Flor-

ence to come to Milan in 1483, the seeds of a scientific career had been truly well sown. Leonardo's position at Milan was that of consulting engineer to Ludovico Sforza. In this capacity he was brought continuously into contact not only with the scientific and technical problems of architecture and of military and civil engineering, but also with a number of illustrious scientific contemporaries of the period. Of these we may mention Fra Luca Pacioli the mathematician, Fazio Cardano the father of the famous Jerome Cardan and a well-known student of perspective; Bramante the famous painter and architect, and Peruzzi the astrologer and mathematician. Near Milan, too, was Pavia, with its university and library and here, it is known, da Vinci spent many an hour delving into the writings of the classics.

Towards the year 1499 political events began to make Leonardo's position at Milan untenable, and in December of that year he set out, accompanied by his friend Pacioli, for Florence. They traveled via Venice and spent their time chiefly in the study of cosmography. Back again in Florence, Leonardo da Vinci continued, as ever, the man of science. Problems of nature continually engrossed him, even to the neglect of his art. Thus he undertakes the commission to paint an altar piece for the Church of the Annunciator, but is too much absorbed in his investigations to go on with it. It was at this period, too that he came into contact with Andrea Sansovino, the artist and student of cosmography. The next few years found him alternating between scientific and engineering problems on the one hand, and incompleted works of art on the other, until in 1507 he returned to Milan as painter and engineer-in-chief to the French King Louis XII. The chief scientific friendship of the subsequent few years was with the famous young anatomist, Marc Antonio della Torre, and

the two must have helped and influenced each other very considerably.

Leonardo was now approaching his sixtieth year, and although full of vigor he began to feel the need for peace and quiet. In the summer of 1515 he accepted an invitation from Francis I of France, Louis XII's successor, to take up his residence in the Castle of St. Cloux, near Amboise. Here he spent the remainder of his days. He died on May 2nd, 1519.

### III. THE CHARACTERISTICS OF THE MANUSCRIPTS

It is peculiarly characteristic of our philosopher that he was always making notes. So fertile was his brain, and so much was there for him to ponder over, that if he was to preserve his thoughts at all he must be ever writing. So we find that the notebooks which have thus far been saved alone contain some thousand of pages of closely written matter, dealing with an encyclopoedic range of topics. It is certain, moreover, that huge as is this collection of notebooks many have been lost, probably never to be recovered. It is a remarkable fact with these notes that Leonardo seems to have set out with the deliberate intention of making it as difficult as possible for them to be deciphered and read. We may summarize the characteristics of his handwriting as follows: (1) He wrote all his notes from right to left instead of from left to right, after the fashion of the Semetic languages; (2) he reversed every letter so that it was only possible properly to read them by holding them up to a mirror; (3) he abbreviated a large number of his words; and (4) he dispensed with as many punctuation marks as possible. These make a very formidable array of difficulties for the would-be reader, but happily,

owing largely to the untiring researches of such men as J. B. Venturi, J. Paul-Richter, G. Piumatti, and C. Ravaisson-Mollien, the world has now available practically the whole of the known manuscripts of our philosopher.

Why should Leonardo have made all these difficulties? The whole tenor of his notes clearly indicates that ultimately he had intended to sort them out and possibly to recast them in a form suitable for publication. Meanwhile, however, there was no doubt a need for protection against plagiarism. Further he had to consider the need for protection against the authority of the Church. He had either to practice restraint in expression or alternatively to protect himself against injudicious publicity. Since, however, nothing could be further from da Vinci's nature than restraint, the need for secrecy was obvious. When we look closely into Leonardo's style of writing, we are soon confronted with the fact that his language was crude and frequently vague. It is easy for the enthusiast to read more into the notes than is justified by the facts. On the other hand it is also possible to err on the side of judging the vaguenesses and crudities of language too harshly. By virtue of the circumstances under which the notebooks were compiled repetitions of both theme and problems were inevitable, and the critic must at all times remember that no philosopher can be finally judged by his casual notes, jotted down at various odd moments. What matters are the connected writings derived from these notes. Now this was a stage that Leonardo, as we have seen, definitely intended, but never actually reached, and this must most scrupulously be allowed for in passing judgment upon him.

It is undeniable that Leonardo suffered badly from the want of a precise scientific vocabulary. Thus in his notes on mechanics, although such modern ideas as force, inertia, work, etc., are constantly to be found, they are cloaked



in a phraseology which is rarely precise, and is frequently puzzling. Yet the notion of rigidity in scientific thought had little place in the fifteenth century. Looseness of expression is frequently the result of ignorance. A wealth of such looseness is not as unknown device as a cloak of assumed wisdom. Yet no one can accuse Leonardo da Vinci of belonging to this class of writer. There is, however, the other side of the picture—the looseness of expression due to a sheer inadequacy of words to convey new ideas such as may occur to a writer who lives ahead of his time. It is to this side of the picture that Leonardo belongs. Many of his ideas, lost with the dispersal of his manuscripts, had to be rediscovered by others long after him. These others lived at a day when scientific terminology was more complete, a fact that should not blind us to the difficult paths of language through which Leonardo had to grope his way. Finally we must remember that clarity of thought in the exact sciences could not properly come into its own until the test of experiment had been recognized and practiced. History records that the pioneers of experimental science were Galileo in Italy and Gilbert in England. Now Leonardo, more than one hundred years before Galileo was most definitely an experimentalist, and granting that the great Tuscan philosopher was indeed the "father of experimental philosophy" then we may claim for da Vinci that he was its grandfather. The appeal to experiment pervades all his writings. Thus we read, "Before making this case a general rule experiment it one, two or three times, and see if the experiments produce the same effect." Small wonder, therefore, that his ideas outstripped his language, and that he was compelled to express himself with a vagueness that was inevitable in the circumstances.

## IV. LEONARDO'S COSMOGRAPHY AND GEOLOGY

We will consider in turn various aspects of Leonardo's scientific activities as evidenced by his notebooks, and in the first instance we will offer a few remarks as to his views on the scheme of the universe. In the early part of this article it was pointed out that the characteristics of mediaeval cosmography were bound up with the geocentric philosophy of Aristotle, with its unchanging and ever-perfect skies, its systems of revolving spheres, and its ever-growing schemes of epi-cycles, each element of which was brought into being to "save the phenomenon." But the spirit of unrest was abroad. Nicholas of Cusa, Toscanelli, Peurbach and Regiomontanus were each in their turn contributing towards the revolution of thought which was to follow. Cusa had pointed out that from an astronomical point of view there is nothing unique about the earth—that it was merely one star amongst so many others. To this point of view Leonardo himself contributed, and in Manuscript F of the collection at the Institute of France not only does he repeat Cusa's contention, but he says further that to an observer from some other star the earth would be seen as a star. He develops also an interesting argument based on considerations of friction. He points out that friction consumes the bodies which are rubbed and he enquires into the effect which the revolvings of the concentric systems of spheres would have, each member rubbing against its neighbor day by day for centuries. "How," he asks, "could they persist without being used up?" He even goes so far as to point out that whereas the linear velocity of rotation is zero at the poles of the

spheres it increases to enormous values at the equator; so that here the friction would also be enormous, and the system could not persist.

Some controversy has been directed towards the question of whether or no Leonardo believed in the diurnal rotation of the earth on its axis. Attention is drawn by one side to a note in Manuscript G in which Leonardo discusses the spiral motion of a body falling towards the rotating earth, and it is difficult to believe that he would have propounded this problem at all if he were not prompted by some such idea as that the earth is revolving. It is, however, a little more difficult to accept the view that Leonardo was a Copernican. We must not read too much into the famous note in the Windsor Manuscript, "Il sole non si muove" (the sun does not move). The notebook in question was not completed later than 1515, whereas the *De Revolutionibus* of Copernicus could not have been completed before 1529, and was not published till 1543.

Leonardo devoted much consideration to both the sun and the moon. Their points of similarity and contrast appealed to him. He speaks of the sun as the king of the universe, the source of all light, heat and life—a view so far contrary to the orthodox outlook of the Church of Rome as to make his methods of secret writing perfectly comprehensible. Leonardo knew that the moon shines by light reflected from the sun, and he entered into interesting speculations as to the existence of continents and seas on the moon's surface. One of his most interesting observations are to be found in the Leicester Manuscript. Here he discusses the existence of a sort of lunar force of gravitation. He argues that the moon being spherical is like the earth, and therefore is dense, and therefore has weight. "How, then," he asks, "can the moon sustain itself in its place in space?" If there is no such thing as a force of

gravitation (Leonardo did not, of course, use this term) on the moon such as prompts the body to fall to itself, as would a stone on the earth, then the seas on the moon, for example, would of necessity fall to the earth. But they do not do this, and so Leonardo concludes that such a tendency exists for the moon in the same way as for the earth.

Passing next to geology we come to another field of interest to our philosopher. Many of Leonardo's notes on this subject are to be found in the Leicester Manuscript, and they read startlingly modern. Thus he discusses the slow carving out of a valley by a river, the persistent erosion of its banks, and the steady deposition of the debris which has been carried towards the river mouth. Leonardo was probably the first scientist to appreciate the real significance of fossil remains. His discussion of these is masterly. He attacks the current theory of the advent of a deluge by pointing out that the geological processes of change on the earth are ever present and tend to be gradual; and that violence in nature is rare and is insufficient to explain the phenomenon of fossils.

There is no time in this brief article to consider in detail the ordered sequence of arguments that Leonardo marshals forth in opposition to the pro-deluge theory. They show, however, a true master of scientific method, and lead our philosopher to the modern theory of the gradual encroachment of sea on land through the ages, and of the submergence of continents.

## V. GENERAL PHYSICS

Turning next to the group of subjects of heat, sound, light, magnetism, etc., collectively referred to nowadays as physics, we come to a wide sphere of activity on the part of Leonardo da Vinci. His notes on these subjects are

scattered very generally through his various notebooks, but as might naturally be expected, the subject which comes in for most of his attention is that of optics. On the other hand, although the earliest references in history to the simple phenomena of electrification by friction carry us back to Thales of Miletus, there is no mention whatever in Leonardo's Manuscripts of this subject.

The references to heat are somewhat scanty, but in any case, very little had been done by philosophers, apart perhaps from Roger Bacon in the thirteenth century, towards serious discussion on this subject. Leonardo appears to have linked up the phenomena of heat with light from the point of view that heat springs from luminosity, and he instances such varied examples as the sun and the flame of a candle. This aspect is considered both in the *Codex Atlanticus* and in Manuscript G of the collection at the Institute of France. The expansion property is evidently referred to in the Ashburnham Manuscript, and in course of which he refers to the "greater solidity of the fluids where there is a greater coldness." His observation is at fault, however, in a sketch which shows a thick metal slab heated in front of a fire and bent inwards through expansion. The back surface being the cooler the expansion must, of course, be outwards.

Although Leonardo may not have understood the theory of convection currents, he certainly knew of their application, and a number of interesting sketches are to be found of various roasting spits—at this time a very popular device. He observes by way of introduction to this that when two equally heavy objects are suspended from two sides of a balance, and one of them be heated, the hot air in rising will carry it upwards and cause the colder body to descend. Leonardo shows how air heated by a fire below rises and causes a suitably vaned wheel at the top

of the chimney to rotate. The meat is suspended from mechanism geared to the vane, and so rotates also.

Leonardo's notes on magnetism are not very plentiful, but are none the less interesting. At this time both the magnetic properties of the lodestone and the directive property of the compass were well known. Leonardo shows in his sketches that he agreed with the current notion that the compass property of a magnet was caused by the attraction of the Pole star. Thus he writes in Manuscript E, "Take a large vessel and fill it with water. In this water place a wooden vessel and in it place a magnet without any other water. The magnet will float like a ship and immediately after its power of attraction will cause it to move towards the Pole star." We may with some confidence ascribe to Leonardo the invention of some scheme of ring suspension for the purpose of maintaining the ship's compass in a horizontal position in spite of the movement of the ship. A sketch in manuscript A also seems to indicate that Leonardo knew of the phenomenon of magnetic dip. One sees a vertical board spiked at the base so that it may be pushed into the ground. At the upper part of the board is a graduated circle carrying a horizontally mounted magnetic needle sketched to suggest an angle of dip. A plumb line is attached to the apparatus.

We come next to Leonardo's notes on sound. They are remarkable chiefly for the suggestions of some sort of relationship between the mechanisms of propagation of sound and light, and also for the references to some conception of wave motion. Of the origin of sound he writes in the Codex Atlanticus that "A note could be called forth in a body by means of a blow, and this blow could only be made through a movement." Leonardo has some idea of the reflection of sound, and both this and its relationship to the same phenomenon in the case of light are well brought out by a series of sketches in the Codex Atlanticus

in connection with the development of the camera obscura. In the first three of these he has a point-source of light, and he traces the reflections of the light rays in varying conditions.

In the succeeding three sketches we see a corresponding scheme for sound rays, the sound being emitted by the striking of a hammer against an anvil. In a note in Manuscript B he definitely asserts the law that the angle of incidence is equal to the angle of reflection, and he shows a sketch of a hammer striking a bell with the resulting sound being reflected from the wall and received by the ear along the reflected ray. From this he passes to the problem of echoes, and he asks the question "If the echo of a voice answers me twice at a distance of thirty yards with two degrees of power of the noise, with how many degrees of power of the noise will it answer me at a distance of one hundred yards?"

There is no doubt that Leonardo appreciated the fact of a spreading out of sound in a spherical form with diminution of intensity. However vague his notes undoubtedly were it is clear he had some idea of a wave motion that was comparable with the spreading out of ripples in circles when a liquid surface is disturbed. Thus we read, "Because in all cases the movement of water has great similarity with that of air, I will connect the example with the above named principles. I say, if you throw two little stones into a lake of still water at the same time and at a little distance from one another, you will see called forth round about the two situated points of contact two separate groups of circles, which groups grow and finally meet each other . . . and the principle of this is that although there is shown an appearance of movement, the water itself is not moved from its place, because the openings which are made by the stones at once close themselves again and cause a certain disturbance to be set up, which



one can speak of rather as a trembling than as a movement."

The bulk of Leonardo's notes on physics refer, as we have already remarked, to the subject of optics. This need not surprise us. Not only does it fit in with Leonardo's role as a painter, but it was the subject above all other branches of physics that had received the greatest amount of attention by his predecessors from Greek times onwards. Yet, prolific as was Leonardo in his notes on optics, it cannot be said that he contributed much that was new. Rather was it a restatement of the ground already covered. The current view of his days, and which he accepted, was that all bodies had certain qualities whereby they were able to transmit their form and peculiarities to the sense organs. Such visible bodies were constantly emitting palpable thin shells from their external visible surfaces, and these met with the visual emanation from the eye. These two, the emanation from the eye and the species, as the palpable thin shells were called, then combined together and passed back to the eye as images.

Leonardo accepts the principle of the rectilinear propagation of light, and as a result he works out a number of elaborate shadow schemes, showing umbra and penumbra. He is familiar, too, with the sort of experiment nowadays referred to as Rumford's photometer experiment. He understood the law of reflection, and was able to apply it to the focal property of a concave mirror. On refraction he was less happy, and his notes on this subject show both vagueness and confusion. Nevertheless he understood the main properties of spherical lenses and spectacles, and his description of the eye clearly evidences the groping after a better theory of vision than was current in his day.

## VI. HYDROSTATICS AND HYDRAULICS

It will be remembered that much of Leonardo's professional work was concerned with structural engineering. He was constantly involved in such operations as drainage and canalization. Consequently it is not surprising to find numerous notes on problems of hydraulics and hydrostatics scattered through his notebooks. His approach to these problems was essentially that of the experimental scientist. Thus he studied the flow of liquids through orifices in considerable detail by carrying out a large number of experiments under such varying conditions as the changing of the speed of flow, and of the size and shape of the cross section of the orifice. From the point of view of theory these investigations did not, perhaps, lead him very far. This we should expect from the nature of the mathematical aids nowadays essential in the theoretical study of this subject, a knowledge of which was then denied to Leonardo. Nevertheless the empirical conclusions at which he was able to arrive were necessarily of tremendous advantage to him in his various engineering operations, and in this connection it is interesting to record that as recently as this year there have been discovered the foundations of the first lock that was built in France by da Vinci in the reign of Francis I. This lock was found near Vierzon, in the Department of Cher, where the famous artist had been sent to map out a route for the salt barges sent from Nantes to Bourges, then the royal residence. The lock was built because it was found that the waterway at Vierzon was not navigable.

Another interesting aspect of Leonardo's activities in applied hydraulics was in his study of water-wheels and water-power. Water-power was becoming increasingly

important in those days. In many technological processes then in vogue, the water-wheel was beginning to do much of the work that was later to be replaced with much greater efficiency by the steam engine. Leonardo's designs of water-wheels show great variety and efficiency.

Actually Leonardo had confined himself to what one might call the artificial aspects of hydraulics. With his breadth of vision he had no difficulty in linking these up with the natural aspects, i. e., with such phenomena as ocean currents, water falls, springs and subterranean waters.

Turning from hydraulics to hydrostatics we come from a field of empirical achievement to one of positive contribution to theoretical knowledge. Leonardo's notes on the laws of liquid pressure are found chiefly in Manuscripts E and J of the collection at the Institute of France. It is quite clear from these that he anticipated by about one hundred and fifty years Pascal's famous dictum that "water finds its own level." His sketches show that he regards the statement as being equally true whatever the shape of the cross section of the different arms may be. He even applies this to the essentially modern experiment of balancing in a U-tube two liquid columns of different density. Da Vinci clearly states that the ratio of the heights of the liquids above the level of the surface of separation is inversely proportional to their densities. Finally it is interesting to note that Leonardo was beginning to grope towards the discovery of the law of the transmissibility of liquid pressures. He has two vessels, one containing water and the other sand, and he asks the question, "What difference is there between that which receives the weight at the bottom, and that which receives it at the sides?"

## VII. LEONARDO'S MECHANICS

Of all the branches of natural science to which Leonardo gave his close attention probably none received more of his time and interest so much as that of mechanics. This subject fascinated him. He felt it dealt with fundamentals. He definitely referred to it as "the most noble of all the sciences." Leonardo was right. The basis of all natural science is the conception of force, and the study of force and its effects is mechanics. Leonardo was quite clear about this, and he gives us a number of notes on the meaning of force. "Nothing whatever can be moved by itself but its motions is effected by another. This other is force." Not only does he here attempt a definition of force, but also of the principle of inertia. Furthermore he extends the conception of inertia to the case of moving bodies. In a note in his manuscript on *The Flight of Birds*, he says, "and because all movement tends to maintenance." Again, Leonardo anticipates the law of reactions, for in the *Codex Atlanticus* we read, "An object offers as much resistance to the air as the air does to the object."

Perhaps one of the most illuminating of Leonardo's notes on mechanics is one in which he deals with falling bodies. It gives us, so to speak, Leonardo's philosophy of falling bodies in a nutshell. "Why does not the weight O remain in its place? It does not remain because it has no resistance. Where will it move to? It will move towards the center of the earth. And why by no other line? Because the weight which is in support falls by the shortest road to the line point which is the center of the world. And why does the weight know how to find it by so short a line? Because it does not depend and does not move about in various directions."

Did Leonardo know of the parallelogram of forces and of velocities? Probably he did not absolutely, but most certainly he knew *something* of it. He at least knew that given two forces or velocities there is a resultant effect intermediate between the two. For example, in his Manuscript on *The Flight of Birds* he discusses the case of a bird which tends to be driven in one direction by the wind, whilst its weight tends to bring it down. He definitely says that "The descent of such a bird will be by the mean line between the two." Curiously enough, Leonardo is also aware of a converse to this. Thus in the course of a discussion on the motion of a weight sliding down an inclined plane, we read, "I say that the weight of the body AB (i. e., vertically downwards) divides its gravities in two aspects, that is according to the line BC (i. e., down the plane) and according to the line MN" (i. e., at right angles to the plane).

A large number of Leonardo's notes on mechanics are concerned with the "Principle of Work." Of course, da Vinci never actually used the term "work" in the modern scientific sense. Nor did he speak specifically of a product of force and distance. Nevertheless he did appreciate the fact of a value in, and a measure of, what we may call the *achievement* of a force. Thus he writes, "If a force carries a weight in a certain time through a definite distance, the same force will carry half the body in the same time through double the distance." There is, in fact, a definite limit to the results of a given effort, and this effort is not alone the question of the magnitude of force, but also of the distance, in any given time, through which it acts. If the one be increased it can only be at the expense of the other. All this, of course, is intimately bound up with the age-old fallacy of perpetual motion. Then, as now, there were many men attempting to achieve the impossible. With these Leonardo had no patience. He un-

derstood quite clearly the limitations implied by the principle of work. "If a wheel is moved by a quantity of water, and if this water is not added to either by flow or by quantity or by a greater fall, the function of this water is finished." With such lucid conceptions we can well understand Leonardo writing the following: "Oh, speculators of perpetual motion, how many vain projects of the like character you have created; go and be the companion of the searchers after gold."

Leonardo pays considerable attention to the study of centers of gravity. This is not unnatural since it was a subject of peculiar importance to him, not only from the point of view of the engineer, but also from the point of view of the artist, since he was constantly concerned with the study of the balance of the human form. Leonardo did not attempt a definition of center of gravity. He evidently took it for granted. It was a conception that had literally grown up through the ages. It was, in fact, a commonplace of ancient and mediaeval science. He did, however, study the subject experimentally. More than once we meet with a sketch of a suspended body with some such note as "The center of all suspended weights is established at its point of support." Or again, "This is done to find the center of gravity of a bird." A variety of sketches, too, show that Leonardo studied the distribution of weight about the vertical line through the center of gravity in such varying circumstances as standing, sitting, kneeling, walking up and down hill, mounting stairs, etc. He appears, too, to have been the first to study mathematically the problem of the center of gravity of the tetrahedron, which he definitely states to be a quarter of the way up the axis.

On the principle of the lever Leonardo writes extensively. This again was a field of study upon which many of his predecessors had worked from classical times on-

wards. Leonardo's contribution, however, was in his reducing the problem of the bent-arm lever to the case of an equivalent simple lever by, in effect, revolving one arm along the direction the other arm produced. He spoke of this equivalent arm as the "potential arm," and he made this the starting point of a whole series of investigations in statics. One of the most thorough of Leonardo's studies was that of the pulley and of pulley systems. His sketches in this direction are very plentiful, and strike one as being forcibly modern. Pulley systems of great variety and of varying orders of simplicity and of complication abound, and they show very clearly how accurate were our philosopher's ideas on the subject of the string tensions and the mechanical advantages in the various cases.

Naturally all these investigations find continual application in hosts of directions. The transmission of power, the designing of machinery of various kinds, the design of toothed wheels and of chains, and of mechanical appliances of almost every conceivable kind—all these are to be found most liberally distributed through his manuscripts. Nor should we omit to mention what was evidently for Leonardo a most absorbing study—that of flight. The problem of air-conquest has ever been a tantalizing study throughout the whole history of man, but we may claim for Leonardo that he was essentially the first to study the problem from a scientific standpoint. This, however, has been dealt with by the writer at considerable length elsewhere, and need not be further discussed in this article.

Looking broadly over Leonardo's mechanical studies, and regarding them from the point of view of the two modern divisions of statics and dynamics, it should be noted that the former come in for a much larger proportion of Leonardo's time than the latter. This, however, is easily explained by the fact that amongst Leonardo's predecessors statics was almost invariably the subject dealt



with, whilst dynamics was left untouched. For the same reason we find that although the notes on dynamics are less plentiful in Leonardo's manuscripts, they are much more original. Where in statics Leonardo was more or less repeating from his own point of view what had been done before, in dynamics he was creating and sowing from virgin soil.

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## VIII. CONCLUSION

We have now passed in review a summary of the main achievements of Leonardo da Vinci in physical science. It is an astonishing record. It must be remembered, moreover, that this represents but a fraction of our philosopher's scientific activities. As physiologist, anatomist, biologist, architect and engineer, there were yet other remarkable achievements which it has not been the purpose of this article to discuss. And in addition to all these there is forever and at all times his incomparable art. Small wonder that Leonardo has been singled out time and again as one of the few of the world's greatest personalities. How strikingly, too, does this bring home to us the colossal tragedy of the loss of his manuscripts to mankind for so many hundreds of years! The story of this dispersal and loss is too long a one to be discussed here. But how differently might the course of scientific history have run if the great pioneers of research who lived after him could have had available to them the fruits of da Vinci's great labors. But this was not to be, and as a result much that he did had to be re-done after him, and only today are we beginning to realize the true extent of the world's great loss.

IVOR B. HART.

UNIVERSITY OF LONDON, ENGLAND.

## THE VITALITY OF LAO-TZE'S PHILOSOPHY

**L**AO-TZE was a profound and original thinker and philosopher of China in ancient times. The date regarding his birth and death is a disputed one, but most probably he was born about 578 B. C., and died eighty years later.

He was a native of Chu or Chen, two states in south China in his time. Just as the nature of southern people is active and progressive, so Lao-Tze's philosophy was rather radical, reactionary and revolutionary. He served as the president of the National Library, and acquired a very profound historical knowledge. In those turbulent times, the land was torn by internecine feuds, and the spirit of militarism was rife just as it is now. It is not surprising that Lao-Tze expressed unqualified abhorrence of war and created a system of quietism. Thus his theory is not fantastic or fallen down from the heaven; but it was based upon facts and came out of the situation.

He embodies his philosophy in his book of five thousand words called *Tao Teh Ching*, the main thesis of which is a discussion of Tao (or Way). Let us first make an inquiry into what is the nature of Tao.

What is the nature of the universe? This is the primary and natural question in philosophy. This problem so far has not been solved. Lao-Tze interpreted the nature of the universe as follows:

"There is a thing that existed before the creation of Heaven and Earth. It is independent without changing and goes around without endangering. It is regarded as the mother of the universe. Its name I

know not. Designating it, I call it Tao. Describing it, I call it Great.

"Heaven takes its laws from Tao. Tao takes its laws from its own spontaneity.

"Tao can not be seen, nor heard, nor touched. It returns to nothingness. It is the form of the formless, the image of the imageless, the fleeting and the indeterminable. Before it, you can not see its head; following it, you can not see its back.

"Tao in itself is vague, impalpable—how impalpable, how vague! Yet within it there is Form. How vague, how impalpable! Yet within it there is substance. How profound, how obscure! Yet within it there is Vital Principle. This principle is the Quintessence of Reality, and out of it comes Truth."

From the above paragraphs we understand what Lao-Tze meant by Tao. The nature of Tao is beyond the conception of Heaven. Lao-Tze did not recognize the old conception that the world was created by God.

As Lao-Tze said that there is a thing before the creation of Heaven and earth, it seemed that the nature of Tao is a concrete thing. However, Lao-Tze feared that there might be misunderstanding about his idea, and therefore immediately warned that Tao can not be seen, nor heard, nor touched. It can not be named, but it returns to nothingness. If this is so, then Tao is nothingness. As Lao-Tze also feared that there might be a possibility of misunderstanding, he asserted without hesitation that "Tao is the form of the formless, the image of the imageless." Again: "How impalpable, how vague! Yet within it there is form. How vague, how impalpable! Yet within it there is substance." Then the question arises: Is the nature of Tao a thing or nothingness? According to Lao-Tze, the question of a thing or nothingness is a question of nominalism. Nominalism should not be applied to the nature of Tao.

How can we understand the nature of Tao? As Buddha indicated, it is inconceivable. It must be understood that it is not that it can not be conceived, but that it should not be conceived. That which involves conceiving is not nature. Lao-Tze said: "Those who know do not say, those

who say do not know. The further you go, the less you know." The nature of Tao can not be understood by means of knowledge or intellect. Also Lao-Tze said:

"When the superior scholar hears of Tao, he diligently practises it. When the average scholar hears of Tao, he sometimes retains it, sometimes loses it. When the inferior scholar hears of Tao, he loudly laughs at it. Were it not thus ridiculed, it would not be worthy of the name of Tao."

As the nature of Tao is inconceivable, we can only proceed from nominalism. The first paragraph of "Tao Teh Ching" was devoted to the relation of nominalism and the nature of Tao. Said Lao-Tze:

"The Tao which can be expressed by words is not the eternal Tao; the name which can be uttered is not its eternal name. Without a name, it is the beginning of Heaven and Earth; with a name, it is the mother of all things. Only one who is eternally free from earthly passions can apprehend its spiritual essence; he who is ever clogged by passions can see no more than its outer form. These two things, the spiritual and the material, though we call them by different names, in their origin are one and the same. This sameness is a mystery—the mystery of mysteries. It is the gate of all spirituality."

The question may be raised: Whence comes the term "name"? It comes, according to Lao-Tze, from the distinction made by human beings. His idea was that if all the world knows that beauty is beauty, then it is ugliness; if all the world knows that good is good, then it is bad. That is, how can we know beauty and good? It can be stated as follows: Because we compare beauty with ugliness, we have the conception of beauty and also at the same time the conception of ugliness. The same can be said of good and bad and also of other things. In his own words:

"Among mankind, the conception of beauty as such implies the idea of ugliness, and the recognition of good implies the idea of evil. There is the same mutual relation between existence and non-existence in the matter of creation; between difficulty and ease in the matter of accomplishing; between long and short in the matter of form; between high and low in the matter of elevation; between treble and base in the matter of musical pitch; between before and after in the matter of priority."

Human beings create names. After the names have been created, they can never be eliminated. Then we interpret everything with these names. Regarding the process of the creation of names, Lao-Tze said:

"Tao produced unity; unity produced duality; duality produced trinity; and trinity produced all existing objects."

How can we explain this process? Unity and duality are one pair. Without duality, the name of unity can not exist. Naturally, unity is connected with duality. Therefore unity produced duality. And unity and duality combined constitute trinity. Trinity is independent and equivalent to one. It must be connected with the relative term "duality." Thus innumerable unity and duality combined produce all existing objects. This explanation can be easily understood with reference to biology. All things are produced by the interaction of positive and negative.

Hence it might be said that Heaven and Earth are the signs of two forces: positive and negative, and also the signs of unity and duality. The interaction of positive and negative, according to Lao-Tze, is like an instrument. Though the instrument itself is empty, it can produce a number of sounds when it assumes activity. This activity is the source of all existing objects. But whence comes the activity? As has been said, "Tao takes its laws from nature." To it can not be added any external knowledge. "Nature" is the nucleus of Lao-Tze's philosophy. Since then naturalism has been in the center of Chinese thought.

Having adequately explained the origin of names, Lao-Tze added that the conception of names is not a good one. He meant that names are derived from man-made distinctions upon which we can hardly rely. In fact, there is no standard of good and evil, loss and gain. During the Great War, for example, Frenchmen wanted to kill all the Germans, and the Germans all Frenchmen and Englishmen.

All of them would claim that what they had done was good, because they loved their own country. Is that really good or not? Lao-Tze said: "Good and evil are not far from each other." To him, the different names of good and evil are all given by human beings; this is contradictory to nature.

Bearing the previous paragraph in mind, we can easily understand why Lao-Tze said as follows:

"When the Great Tao falls into disuse, benevolence and righteousness come into vogue. When shrewdness and sagacity appear, great hypocrisy prevails. It is when the bonds of kinship are out of joint that filial piety and paternal affection begin. It is when the State is in a ferment of revolution that royal patriots arise.

"As restrictions and prohibitions are multiplied in the Empire, the people grow poorer and poorer. When the people are subjected to overmuch government, the land is thrown into confusion. When the people are skilled in many cunning arts, strange are the objects of luxury that appear. The greater the number of the laws and enactments, the more thieves and robbers there will be.

"Cast off your holiness, rid yourself of sagacity, and the people will benefit an hundredfold. Discard benevolence and abolish righteousness, and the people will return to filial piety and paternal love. Renounce your scheming and abandon gain, and thieves and robbers will disappear. These three precepts mean that outward show is insufficient, and therefore they bid us be true to our proper nature—to show simplicity, to embrace plain dealing, to reduce selfishness, to moderate desire."

Again he said:

"That we have calamity is due to our conception of self-existence; as soon as we renounce the conception of self-existence, there will be no calamity."

In a word, to return to nature it is only necessary to give up the conception of distinction and get rid of selfishness.

Lao-Tze's book, or rather the book attributed to him, consists of five thousand words of which four thousand are devoted to the description of the operation of Tao. One sentence which can comprise the whole idea is: "Tao is eternally inactive, and yet it leaves nothing undone."

It is true that Lao-Tze advocated inaction; nevertheless he wished to leave nothing undone, as has just been said. Let us quote from him:



"The sage occupies himself with inaction, and conveys instruction without words—production without possession, action without self-assertion, development without domination: this is the mysterious operation of Tao."

This is not only extraordinarily interesting but has an everlasting merit and absolute truth. Human beings possess two kinds of impulse: possessive and creative. The first takes possession of something as private property. This kind of property is limited. In economic interest, for example, if A takes more, B and C are bound to gain less. The same can be true with the political power. Were the possessive impulse developed, mankind would engage in a fierce struggle. This kind of impulse, in view of everything, must be diminished as much as possible. The creative impulse, oppositely, is to create something and let everyone enjoy it. The nature of this kind of thing consists in being public and limitless. Philosophy, science, music, literature and aesthetics all belong to this category. The more you create, the more mankind will enjoy.

Thus the creative impulse is toward helping others. Only he who can comprehend Tao can contribute to mankind. In encouraging the spirit of serving to mankind, Lao-Tze said:

"The sage does not care to hoard. The more he uses for the benefit of others, the more he possesses himself. The more he gives to his fellowmen, the more he has of his own.

"When merit has been achieved, do not take it to yourself; for if you do not take it to yourself, it will never be taken from you.

"The sage expects no recognition for what he does; he achieves merit but does not take it to himself; he does not wish to display his worth.

"It is the way of Heaven to take from those who have too much, and give to those who have too little. But the way of man is not so. He takes away from those who have too little, to add to his own superabundance. What man is there that can take of his own superabundance and give it to mankind? Only he who possesses Tao."

This passage is equally true. The more money you give to others, the less you have. It is because this kind of prop-

erty belongs to possessive impulse. The creative impulse is just the reverse. The great scholars bestowed upon us so much knowledge, yet they themselves did not have a bit of loss. If you draw a very good picture and let others see it or compose a piece of music and let others hear it, others will appreciate your creative merit and your knowledge can also be improved as well as increased. This is what Lao-Tze said: "The more he uses for the benefit of others, the more he possesses himself. The more he gives to his fellowmen, the more he has of his own." One must expect return from others.

Realizing that the world was always in a state of terrible strife, Lao-Tze said:

"The highest goodness is like water, for water is excellent in benefiting all things, and it does not strive.

"It is the Way of Heaven not to strive, and yet it knows how to overcome . . .

"It does not strive, therefore it has no sin.

"The reason why rivers and seas are able to be lords over a hundred mountain-streams, is that they know how to keep below them. That is why they are able to reign over all the mountain-streams.

"He who knows others is clever, but he who knows himself is enlightened. He who overcomes others is strong, but he who overcomes himself is mightier still."

But how can we make all the people not strive? The most important thing is to understand the principle of non-possession. Lao-Tze said:

"Heaven and Earth are long-lasting. The reason why Heaven and Earth can last long is that they live not for themselves, and thus they are able to endure."

Lao-Tze meant encouraging mankind to neglect self-interest. It is only by getting rid of selfishness and by breaking up of the conception of possession that strife will not exist. He had the same idea in advocating "return to nothingness."

Here we can not help associating Lao-Tze's philosophy with Darwin's *Theory of Evolution*. Darwin's principle

called forth a revolution in the world of thought, and it is fair to say that it contributed much to the field of knowledge. But, afterwards, the principle of "survival of the fittest" was applied to mankind; such application had done more harm than good to humanity as a whole. The Great War served as a good example. Though the causes are many, Darwins' theory has indeed great influence. Lao-Tze's philosophy has of late gained a favorable ground in Europe, which is probably a reaction toward Darwin's theory.

Lao-Tze held that one should take nature as the foundation of his action which must not be contrary to natural law. Therefore, he always referred to natural phenomena. For example, "this is the Way of Heaven, which benefits, and injures not"; "it is the Way of Heaven to take from those who have too much, and give to those who have too little." Among mankind, Lao-Tze held a child is most near to the natural mode, and we should learn from him. What is the mode of the child? Lao-Tze said:

"He who trusts to his abundance of natural virtue is like an infant newly born, whom venomous reptiles will not sting, wild beasts will not seize, birds of prey will not strike. The infant's bones are weak, its sinews are soft, yet its grasp is firm. All day long it will cry without its voice becoming hoarse. This is because the harmony of its bodily system is perfect."

A child is innocent and does something without any conception of compensation. This is just unselfish action. If Confucius, Socrates or other great scholars were ever conscious of compensation, we could never have such valuable philosophy and culture.

As Lao-Tze advocated, "Tao takes its laws from nature," he wished to imitate it and apply this principle to politics. He said:

"Were I ruler of a little state with a small population, and only ten or a hundred men available as soldiers, I would not use them. I would have the people look on death as a grievous thing, and they should travel

to distant countries. Though they might possess boats and carriages, they should have no occasion to ride in them. Though they might own weapons and armour, they should have no need to use them. I would make the people return to the use of knotted cords.<sup>1</sup> They should find their food sweet, their rough garments fine. They should be content with their homes, and happy in their simple ways. If a neighboring state was within sight of mine—nay, if we were close enough to hear the crowing of each other's cocks and the barking of each other's dogs—the two peoples should grow old and die without there ever having been any mutual intercourse."

Such is Lao-Tze's Utopia. But it could by no means be realized.

In regard to study, let us quote from him:

"The pursuit of book-learning brings about daily increase. The practice of Tao brings about daily loss. Repeat this loss again and again, and you arrive at inaction. Practise inaction, and there is nothing which can not be done."

Lao-Tze meant that if you want to get knowledge, you must add something from the external world; if you want to cultivate your mind, you must diminish all the external things. What is the basis of this theory? Lao-Tze said:

"A variety of colors makes man's eye blind; a diversity of sounds makes man's ear deaf; a mixture of flavors makes man's palate dull."

I think this passage is quite right. In modern times, we have electricity; consequently, those who have the trouble of being short-sighted are increasing in number. The same is true in regard to sound and flavor. Many Americans and Europeans are becoming nervous wrecks—due, largely, to the increasing use of tobacco. The more stimulating our literature and music, the more popular it becomes. All these factors may be held responsible for much of the nerve tension of today. This is the pathological phenomenon of society.

Therefore Lao-Tze encouraged living a simple life. The more you diminish your sensational desires, the more your mind will be clear. Lao-Tze said:

<sup>1</sup> The old quipo method of recording events before the invention of writing.

"There is no sin greater than ambition; no calamity greater than discontent; no vice more sickening than covetousness. He who is content always has enough."

All selfish motives arise out of the "possessive impulse." This is what Lao-Tze called "covetousness." Since you are always covetous, you can never be content. How can you work for society or the world? Thus, to diminish the sensational and selfish desires, as Lao-Tze taught us, is not negative but positive. Let us repeat what Lao-Tze said:

"He who knows others is clever, but he who knows himself is enlightened. He who overcomes others is strong, but he who overcomes himself is mightier still."

Lao-Tze's great ambition was to create a systematic philosophy and give us many problems to study. His philosophy is a human philosophy. And his political lesson of Laisser-faire which the Chinese people have well assimilated has also great value. "From two great evils of modern civilization—the bane of over-legislation and the pest of meddlesome and overbearing officialdom—China is remarkably free." Certainly, the Chinese people enjoy an absolute liberty of action.

C. F. LIU.

## CRITICISMS AND DISCUSSIONS

### CARMICHAEL'S REPLY TO KLYCE

To the Editor of THE MONIST:

It was rather interesting, and sometimes a little amusing, to read the article, "Foundations of Mathematics," on the proof sheets which you kindly sent to me some days ago. In the opening paragraph of that article the author indicates its purposes in the following words:

(1) "I shall show that Professor Carmichael's fundamental mathematics in the October, 1923, *Monist* (pp. 513-55) is unsound or illogical, or is non-mathematical in any self-consistent sense of the word *mathematical*."

(2) "I shall show . . . that . . . orthodox mathematical foundations are unsound."

(3) "I shall establish sound mathematical foundations."

If the author had convinced me of his success with the first purpose, I would be very grateful to him. What I found, however, were some criticisms of my article for not doing what I had no purpose to undertake doing in the article, and some other futile criticisms relating to the actual content of the article. In no case have his arguments convinced me that any part of my paper needs change on account of his objections nor that I should now write another paper to cover the things which I did not treat in the former. Moreover, the artillery-fire directed against myself and all other mathematicians is such as not to call for any counter-fire, since it leaves our fortress entirely uninjured. There are some rather furious discharges, but they dissipate themselves without taking effect. Since they do not take effect I can see no need of giving further attention to them.

As to the "mathematical foundations" which are attempted in the third part of the paper, I may say that they are of no use to me as a mathematician. It is difficult to see how anyone could make an effort in this direction and go so far wide of the mark—if he has any adequate knowledge of mathematics at all. Others may be

helped in determining whether they wish to read this part of the article by noting the following isolated quotations from it: "Mathematics obviously never will say anything so long as it confines its language to the elementary, primitive clucks of a hen or the gestures of an earth-worm." "Or, in soundly consistent language, there are *two* verbal contradictions, which obviously cancel each other. And that is the total mechanism of sound mathematics or language." "So until we learn some direct method of repairing defective brains, there is no positive or direct way of teaching sound mathematics." Orthodox mathematics omits both the real proof and the logical proof—and thus both practically and logically has no base, no foundation, and *strictly* no sense."

No further comment seems necessary.

R. D. CARMICHAEL.

URBANA, ILLINOIS.



## KLYCE'S REBUTTAL

TO THE EDITOR OF *The Monist*:

My article in the October, 1924, *Monist*, which Professor Carmichael dogmatically pronounces worthless and irrelevant, amounts to this:

Carmichael, in his article, gave a typical orthodox statement of a mathematical foundation, which he claimed was sound (i. e., contained no self-contradictions). I showed that it actually was full of contradictions—that like all orthodox mathematics it basically took *class* and *element-of-a-class* to be *logically the same*, whereas according to our accepted dictionaries and the most elementary observation or common-sense, a class (set, aggregate, One) is a collection; and an element (number, Many “thing”) is *not* a collection, but is the *logical opposite* or “contradiction” of “collection.” In short, his typical mathematics played fast and loose with our language or dictionary, the words of which however he *tacitly claimed* to be using in their usual senses; or, in general, he ignored the problem of class-element, or One-Many, and inevitably was full of contradictions (many of which I pointed out in detail). I then gave the solution of that logical problem of the One-Many, and showed that the solution was actually the foundation of *sound* mathematics, and was the foundation mathematics themselves *intuitively* used whenever they got anything right. And mathematicians *usually* (by such *intuition*) get applied mathematics right, but only as a rare accident get their asserted foundation and alleged “rules” or logic right (i. e., self-consistent, or in *steadily* honest agreement with the dictionary they *tacitly claim* to be using).

Carmichael replies to my article to this effect: “I am able to recognize mathematical sense and relevance. I see neither in Klyce’s article. Therefore, there is neither in Klyce’s article.”

He in effect adds the incidental comment that I (Klyce) have no "adequate knowledge of mathematics at all." Finally, as a casual afterthought he tries to drag an odoriferous red herring across the trail of his dogmatic *ipse dixit*, by giving four isolated quotations from my article, not one of which is intelligible to anybody who doesn't know the context, and at least two of which have a positively distorted meaning if it be attempted to understand them in isolation. It is charitable to judge that Carmichael is so innocent that he didn't recognize the nature of what he was doing by such quoting. So in order to show him what can be accomplished by isolated quotation, I here quote in isolation one statement of his own article:

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"BCDEFGA

"DEFGABC"

If you tried to interpret that remark in isolation, your only reasonable conclusion would be that he was gibbering—and I assure you he wasn't quite doing that.

So the gist of Carmichael's reply is his dogmatic assertion that he can see no sense and relevance in my article, and therefore there isn't any there. He thus evades discussion of my argument—but by ex-cathedra assertion that I am a logical heretic, merely raises the question of what actually are the views of authorities in such matters. I did not go into that point in my article—simply giving in it the essential facts and arguments. But since Carmichael now premises that he is infallible authority, it will be interesting and profitable to your readers to see some typical facts on that additional, somewhat secondary, point.

My article (as was indicated in it) is a condensation, from the logical or mathematical point of view, of my book *Universe*, recently published. That book, as Carmichael well knew, contains approving Introductions by John Dewey and David Starr Jordan. (E. g., Dewey speaks of the book as "remarkable, noteworthy," and as having "extraordinary value"; and Jordan, as "unique," "daring," "successful"; and both offered evidence in support of their views). Dewey of course is recognized as a leading logician. Jordan many years ago wrote a book (*Stability of Truth*), anticipating James's authoritative logical views. Both Dewey and Jordan obviously have wider reputations for general authority than Carmichael

has; and also have some authority even in logic, the base of mathematics. Hence, Carmichael's dogmatic contradiction of their Introductions is (to say the least) unconvincing, and of extremely doubtful propriety.

Before dealing *directly* with Carmichael's premised authority by quoting mathematical authorities, I may most briefly notice three or four more non-mathematical authorities who conflict with his dogma. (1) Dean T. V. Smith, of the psychological and philosophical department in the University of Chicago, who has written on logic in *The Monist*, concludes in a review of my *Universe* (*American Review*, Jan.-Feb., 1924), that so far as he knows it is the best book on language or logic in existence. (2) Carmichael quotes a psychological sentence from my article as showing how bad the article is; G. Stanley Hall (*Ped. Sem.*, Dec., 1923) says the psychology in *Universe* is "illuminating"; and Dewey in his Introduction speaks highly of it. So we have Carmichael going outside his field, and dogmatically conflicting with highest authorities about my psychological views. (3) I could quote specific approval of the article itself by non-mathematical scientists having as much authority as Carmichael. But I take space for only one more fact not directly mathematical, as it is of wide importance.

(4) My article was printed in *The Monist*, which has for years enjoyed high reputation in the general field of basic mathematics. Carmichael's dogma necessarily implies that the Editors, and whatever advisers they may have had for my article, irresponsibly printed irrelevant trash. By all accepted codes of ethics, a glaring implication of that sort ought to be accompanied by at least a little specific proof of its truth. Of course, I showed that Carmichael's orthodox mathematics was unsound, and hence similarly implied that the Editors printed poor stuff that time. *But*, I gave careful proof in detail. Also, all editors are justified in printing even trash (and ordinarily have to, and do), when it comes from an orthodox authority like Carmichael. (If an authority somewhat obviously goes crazy, we particularly wish to have his own words before our eyes as evidence.) So it is not only editorially proper, but actually desirable, to print even such dogma as Carmichael's letter above; because Carmichael still nominally holds high official authority among mathematicians. But, as ordinary common-sense, which Carmichael ought to know, no reputable editor would, or justifiably could, print an article from an unknown man like me unless it was obviously good in most respects. And I may pertinently add, that

I did not know even the name of any Editor of *The Monist* when I submitted my article.

Whitehead and Russell hold (*Ency. Brit.*, XVII, 881) that (1) mathematicians do not solve the problem of the One-Many, or class-element; that (2) orthodoxly recognized contradictions do occur in mathematics as a result [in my article I simply showed some of the practical ones *typically* made by Carmichael]; and (3) they there try to solve the problem by various evasions of it (and may possibly think they do solve it; they are extremely vague as to that). However, Russell's approval of Wittgenstein's book, which concludes (Prop. 6.54) that everything orthodox mathematics can consistently say about basic mathematics is strictly or logically senseless [as, in fact, it is—as my article proves], shows that he (Russell) has decided that he has *not* solved the One-Many, or class-element.

Thus Carmichael's dogma flatly contradicts and ignores the views of far more authoritative mathematicians than himself: Russell and Whitehead. The charitable conclusion is, that Carmichael is ignorant even of those commonplace mathematical fundamentals (in the cautious, and well-known *Brittanica*) when he dogmatically asserts that my solution of the One-Many is of "no use" to him "as a mathematician." According to Russell and Whitehead's view just quoted, Carmichael doesn't even know that he *doesn't* know fundamental mathematics, and is blissfully unaware that all the *detailed* contradictions I pointed out as occurring in his article have been pointed out *in general* by eminent mathematicians long ago. So as a necessary conclusion from that, Carmichael is quite incompetent to form a judgment as to whether I or anybody else (including himself) has an "adequate knowledge of mathematics."

Immediately upon receiving the proof-sheet of Carmichael's reply, I sent it to Professor William Benjamin Smith, asking him to make a statement I could quote in my rebuttal. In several letters to Smith, during about a month, I pointed out the worst contradictions in the various statements he made. He finally made this statement, as being what he actually meant: "The Many-One problem is fundamental and defiant. I do not pretend to have plumbed its depths, but I am rarely unconscious of its presence. . . . Your own view I have thus far had no opportunity to study."

In my article (pp. 621-2) I showed that Smith published a guessed-at mathematical solution of the One-Many, and I there pointed out its most glaring defects. Apparently (perhaps for that and other reasons) Smith was most reluctant to express any opinion

for my rebuttal, but recognized his professional duty as a mathematician to do so. As a result, he finally in plain effect admits that he considers the One-Many fundamental, considers it to be unsolved by him and hasn't been able to study my view enough to risk a public judgment as to whether I solve it. All of that view of Smith's obviously flatly disagrees with the main points of Carmichael's dogma. It further is a public admission that he, Smith [like Russell and Whitehead], can *not* solve the admitted basic problem in mathematics, and hence, as a direct truism, is incompetent in mathematics.

It happened that I published a review of Wittgenstein's *Tractatus Logico-Philosophicus*, along with a review of Millikan's *Science and Religion* (*Am. Rev.*, March-April, 1924), proving the same general conclusions as my *Monist* article, and the same solution of the One-Many. Professor J. W. Young, one of the authors of the fundamental mathematics Carmichael was quoting as the base of his article, saw my review, and commented frankly on it in a letter to a scientist who then showed it to me.

As soon as I got the proof-sheet of Carmichael's dogma, I asked Professor Young for permission to quote his letter about my review, and for a statement on my *Monist* article which I could use in my rebuttal. Professor Young recognized at once his professional obligation to publish his mathematical views whenever they were pertinent, and with a proper sense of professional duty or ethics gave me permission to quote them.

This paragraph summarizes Young's long letter about my Wittgenstein review. He approved my *general* argument, and accepted my general solution of the One-Many. But he held that "Klyce has adopted an extreme position regarding the nature of . . . 'orthodox' mathematics, which is held by a very small school of mathematical logicians of which Bertrand Russell and Whitehead are the most conspicuous proponents." He says further, "Wittgenstein is not a mathematician at all." So Young concludes that in my review, I *mathematically* am fighting a "straw man"; and that if I only knew a little genuine mathematics, I should agree with him (Young) that my correct solution of the One-Many agrees precisely with the *postulational* methods which really orthodox sound mathematicians [like Young] use.

I of course was aware that perhaps the majority of mathematicians do not accept Russell, etc., as orthodox or sound (although that majority don't publish such a view much, or clearly). So

months before seeing Young's letter, which I summarized in the last paragraph, I wrote my *Monist* article, showing that the *postulational* methods which Carmichael copied from Young were essentially identical with the views of Russell and Whitehead and Wittgenstein (only not nearly so clearly and ably expressed), and were just as wrong—and wrong for the same basic reason: failure to solve the One-Many. Indeed, Whitehead himself uses those same "postulational" methods (*Ency. Brit.*, Art. *Geometry*, Part VII, pp. 730ff)—a mathematical fact of which Young seems to be ignorant. However, Whitehead, with a genuine knowledge of mathematics and its literature, ably points out (l. c., p. 730) that for many years there have been controversies as to the soundness of such methods: that as the actual fact, mathematicians admittedly do not know what a postulate *is*, the answer being the problem of the One-Many. In my *Monist* article I merely showed the practical contradictions Carmichael (following Young) fell into with such methods—and then solved those ancient controversies, all of which obviously amount to the One-Many question.

Young *now*, as the pertinent sum of his several letters to me, says he is *unable* to make any sense out of my *Monist* article, that he doesn't understand my foundation in it [my solution of the One-Many, which is the same solution he accepted in the Wittgenstein review], and that he is convinced that my *Monist* article will be "of no value to mathematicians" because they will "be unable to make sense out of it." Young repeatedly adds that I am ignorant of real mathematics—which is the one point upon which he has agreed with Carmichael's dogmatic reply. But as Young insists that he can't understand my mathematical article, it necessarily follows that he is incompetent to judge my competence.

In short, Young says that no mathematician can understand my article. That contradicts Carmichael, who by dogmatically asserting that it is wrong and irrelevant, necessarily asserts that he understands it and so judges it.

Also, by asserting that mathematicians are unable to understand my article, especially the solution of the One-Many (which he himself had previously said was equivalent to postulational mathematics, although we have seen that Whitehead shows that no mathematician even knows what a postulate is), Young necessarily asserts that he and all other mathematicians are professionally incompetent. That agrees with Russell's, Whitehead's, and Smith's equivalent admission of professional incompetence.

Personally, I *hope* some mathematicians are competent. But I must regretfully admit that I have been looking for some competent mathematicians for fifteen years, and so far have found only one who is basically competent. I don't mean myself: I don't pretend to be a mathematician. I am merely doing the mathematicians' foundation work for them, because they are unable to do it themselves, and are making trouble for all the rest of us by massing up the whole base of knowledge with their clumsy muddling. For years I humbly offered to work with them, and help. But practically all of them, just as Carmichael does in his dogma, for years said in effect, "Heavens! the man doesn't think our mathematics is divinely right: he's a damned fool: throw him out!" So now I have to try another way of establishing sound mathematical foundations.

I also asked Professor George D. Birkhoff to comment on my *Monist* article, and Carmichael's dogma, for my rebuttal. Birkhoff wrote: "My own position concerning the questions involved in your article and in Carmichael's seems to me to be more or less given in the [Birkhoff's] last Lowell Lecture published in *The Scientific Monthly* for August, 1924. You will find it stated there that I am not committed to any particular type of reality as the only valid type . . . thought goes on freely at the mathematical level without any reference back to . . . experience. . . . If I were to attempt to take sides in this matter, I should seem to myself to be leaving solid ground and entering upon metaphysical speculation [compare that with Birkhoff's "mathematical level," which he has just held has absolutely nothing to do with "experience"—with any such vulgar thing as a dictionary or fact]. I do not see that any substantial question is involved, nor can I understand in what way you have added anything on the mathematical side to what is called the foundations of mathematics." In a subsequent letter Birkhoff states: "You will not be justified in stating that I substantially agree with you, for it was only on the side of your analysis of mathematics as a language that I really thought you were going in the right direction. I never have embarked in public discussions and don't propose to begin now."

Birkhoff thus seems to hold, if he holds anything, (1) that he doesn't see that Carmichael's side is any better than mine, both being "metaphysical" rather than mathematical [Birkhoff seems to be a relativist]; and (2) that he can't understand that the problem of the One-Many has anything to do with mathematics. That last



amounts to holding that either Whitehead, Russell, etc. (who think it fundamental), are professionally incompetent; or that he (Birkhoff) is. My article shows that Whitehead, Russell, W. B. Smith, etc., are competent in considering the One-Many fundamental, but are incompetent in being unable to solve it—or even to get it right *in practice* in *basic* mathematics. Birkhoff is therefore incompetent in greater degree. *But*, he says my treatment of mathematics as a language is good mathematics. That obviously is contrary to Carmichael's dogma, and also contrary to his own view that the One-Many adds "nothing on the mathematical side." In short, if his remark about language means anything, he does say that somehow the One-Many is fundamental.

Professor O. Veblen was the other author of the mathematical fundamentals Carmichael quoted as the base of his article. So as the last mathematician, immediately upon receiving the proof-sheet of Carmichael's dogma, I wrote to Veblen, asking for his view. I took the precaution to register my letter to him, and I have the signed "return receipt" for it. But as I have got no reply, I judge that Veblen has so far seen fit to ignore whatever obligation he may have in the matter. Your readers can see that the *three* other mathematicians considered themselves as having a professional duty in this matter, *and met it*—unpleasant as it was to them in some ways.

Not counting the Editors, I have quoted ten authorities. Jordan, Dewey, T. V. Smith, and Hall are non-mathematicians (although three have authority in logic—and this question is logic); and, as has appeared, the four are flatly against Carmichael's dogma in every point they touch in common.

Out of the six mathematicians (Russell, Whitehead, W. B. Smith, Young, Birkhoff, and Veblen), Veblen is a blank in this matter. And so far as I can judge, not one of the other five supports Carmichael's dogma in any substantial point—except some hold that I am a fool because I question the divine infallibility of orthodox mathematics. I have summarized their views above, in as fair a way as I can, and the reader may judge the evidence for himself. In my judgment, that evidence discredits Carmichael in a grave degree.

But Carmichael's competence and professional ethics, if you agree slightly with my judgment of the evidence, has become to us practically a negligible matter. What is important to us, is that the evidence given above has been clearly to the *general* effect that mathematicians are themselves in basic disagreement as to what is

fundamental in mathematics—indeed, as to what mathematics really is, and if there *can* be true mathematics that has anything to do with our “experience” or lives. Also, many mathematicians assert their inability to understand various mathematical matters—which amounts to public confession of professional incompetence.

In view of that definitely proved existence of such a highly unsatisfactory condition in mathematics, I challenge any and all mathematicians to show, by *specific* argument and facts, any logical or mathematical error in my article.

I respectfully and earnestly submit that unless mathematicians can show such substantial error, then if they do not accept *and use* that article as sound, they have discredited orthodox mathematics and themselves—and, if they are honest men, necessarily must resign from their chairs of mathematics until they have corrected their confessed incompetence to fill them. If they find the article too short to be readily grasped in detail, then my lengthy book is available (it is already in many university libraries)—as was definitely indicated in the article itself. However, I have found by trial, that scientists who are not professional mathematicians, or too much befuddled by reliance upon orthodox mathematics, find the article especially clear.

S. KLYCE.

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